(D01) 30 Years of Exoplanets **[90 marks]**

This problem explores some aspects of the two main methods of exoplanet detection: radial velocity and transit. Throughout this problem we shall consider a particular system of a single planet (P) in a circular orbit with radius a around a solar-type star (S). We shall refer to this system as the "SP system".

[Θα εξετάσετε τις 2 μεθόδους εύρεσης εξωπλανητών, ακτινικές ταχύτητες και διάβαση πλανήτη]

(D01.1) The V-band apparent magnitude of the star S is 7.65 ± 0.03 mag, the parallax is 20.67 ± 0.05 milliarcsecond and the bolometric correction (BC) is –0.0650 mag. Thus the star has a brighter bolometric magnitude.

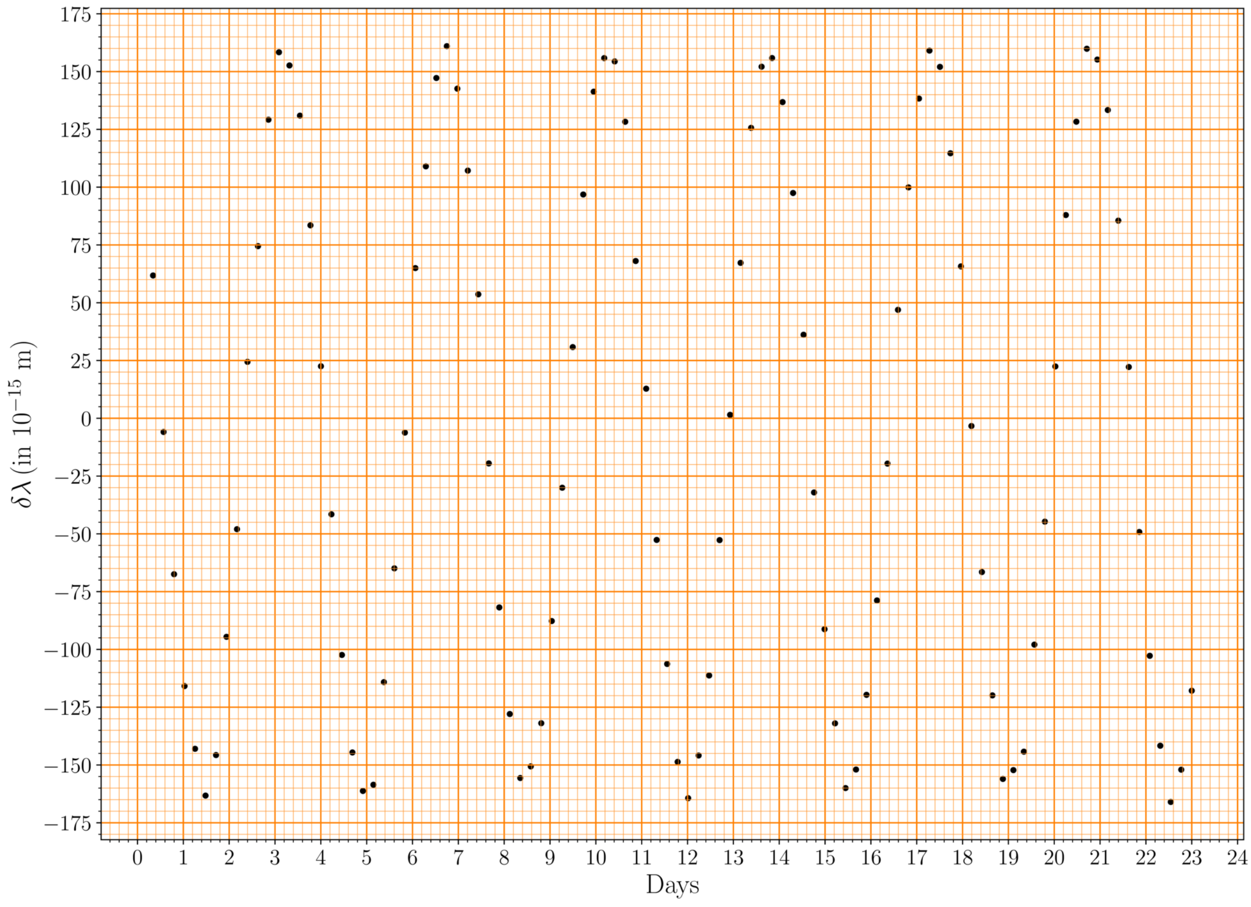
Estimate the mass of the star, M_s (in units of $M_\odot$) , assuming a mass - luminosity (M–L) relation of the form $L \propto M^4$. Also estimate the uncertainty in M_s. You may need $\mathrm{d}\ln x/\mathrm{d}x = 1/x$. **[8]**

[Υπολογίστε την μάζα του αστέρα με την αναλογία που σας δίνεται, και το σφάλμα με την κατάλληλη μέθοδο]

Radial Velocity method

The radial velocity method uses the Doppler shift $\delta\lambda \equiv \lambda_{\rm obs} - \lambda_0$ between the observed wavelength $\lambda_{\rm obs}$ and the rest wavelength λ_0 of a known spectral line to detect an exoplanet and determine its characteristics.

The figure below shows the $\delta\lambda$ for the Fe I line (λ_0 =543.45 × 10^{-9} m) as a function of time as observed for the SP system.



The radial velocity semi-amplitude K is defined as $K \equiv (v_{\rm r,\,max} - v_{\rm r,\,min})/2$ where $v_{\rm r,\,max}$ and $v_{\rm r,\,min}$ are the maximum and minimum radial velocities, respectively.

For a circular planetary orbit the semi-amplitude K can be written as:

$$K = \left(\frac{2\pi G}{T}\right)^{1/3} \frac{M_{\rm p} \sin i}{(M_{\rm p}+M_{\rm s})^{2/3}}$$

where T is the period, i is the inclination of the planetary orbit (angle between the normal to the orbital plane of the planet and the line of sight of the observer), $M_{\rm p}$ and $M_{\rm s}$ are the masses of the planet and the star, respectively.

(D01.2) Use the above graph given in the Summary Answersheet (rotated by 90 deg) to answer the following.

(D01.2a) Draw a smooth curve associated with the observed data shown in the graph. **[2]**

(D01.2b) Select appropriate points on your drawn curve and use suitable methods to determine T and K along with respective uncertainties. All data points used for the calculation of T and K must be shown in the table in the Summary Answersheet. Use the rest of the Table to show your intermediate calculations, as needed, with appropriate headers. **[11]**

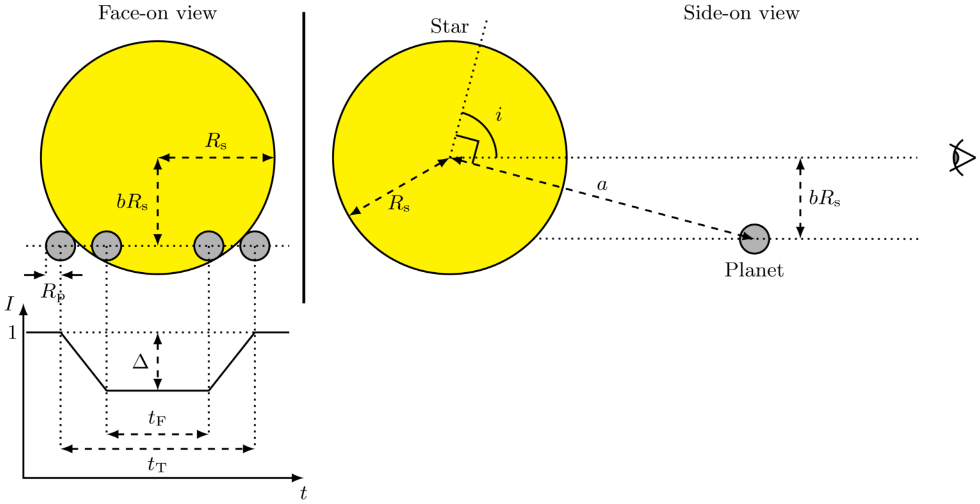
[Πάρτε διάφορα σημεία από το γράφημα, και χρησιμοποιήστε τα για να μετρήσετε τα T και K. Καταγράψετε στον πίνακα όλα τα σημεία που πήρατε καθώς και τις ενδιάμεσες μετρήσεις/υπολογισμούς σας]

(D01.2c) Find the minimum mass of the planet $M_{\rm p,\,min}$ (in $M_\odot$), and its corresponding uncertainty assuming $M_{\rm p} \ll M_{\rm s}$. **[5]**

(D01.2d) Using the value of $M_{\rm p,\,min}$ estimated in part (D01.2c), calculate the minimum value of the semi-major axis of the planet's orbit, $a_{\rm min}$, in au and its uncertainty. **[4]**

Transit method (without limb darkening)

The schematic diagram of a planet transit (not drawn to scale) is shown below. Initially, we shall assume the stellar disk to have a uniform average intensity with some intrinsic noise due to the star itself.



The lightcurve of the normalized intensity, I, as a function of time t is shown in the schematic diagram of the transit above. The average stellar intensity outside the transit is taken as unity. The maximum decrease in the intensity is given by Δ in the normalized light curve. For a uniformly bright stellar disk, the radius of the planet, $R_{\rm p}$, is related to Δ as

$$\left(\frac{R_{\rm p}}{R_{\rm s}}\right)^2 = \Delta,$$

where $R_{\rm s}$ is the radius of the star.

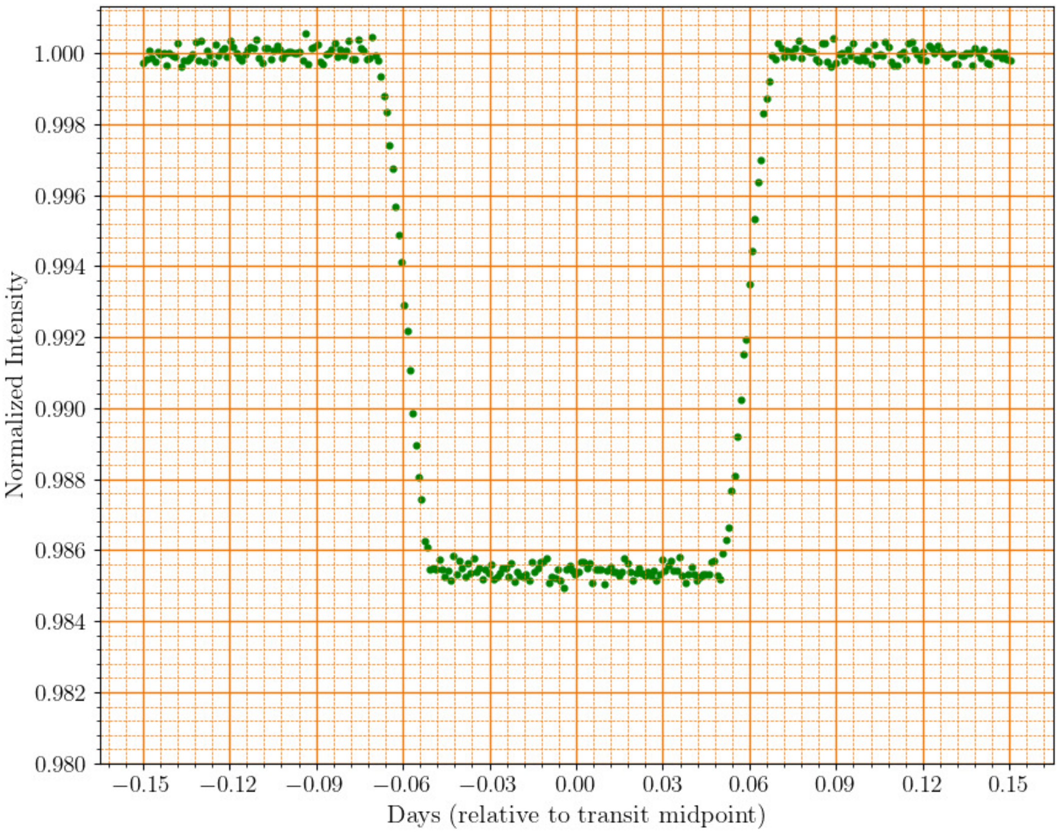
The total duration of transit (when part or all of the planet covers the stellar disk) is given by t_T, while t_F gives the duration when the planet is fully in front of the stellar disk. The "impact parameter" b is the projected distance between the planet and centre of the stellar disk at the mid-point of the transit, in units of the stellar radius, R_s.

For a nearly edge-on star-planet orbit, the impact parameter is given by the formula

$$b = \left[\frac{(1-\sqrt{\Delta})^2 - (t_F/t_T)^2(1+\sqrt{\Delta})^2}{1-(t_F/t_T)^2}\right]^{1/2}$$

(D01.3) For the SP system, the stellar radius is known to be $R_s = 1.20 R_\odot$, and the transit of the planet is indeed visible. Using the minimum orbital radius, a_{min}, estimated in part (D01.2d), find the minimum value, i_{min}, of the inclination angle. **[3]**

Assuming a stellar disk of uniform brightness, the transit lightcurve would look like as shown below.



(D01.4) Using the given lightcurve answer the following questions. For your reference the above lightcurve is also given in the Summary Answersheet.

(D01.4a) Estimate the values of t_T and t_F in days by marking appropriate readings on the graph. **[3]**

(D01.4b) Estimate the mean value of by marking appropriate readings on the graph and hence find R_p in units of $R_\odot$. **[2]**

(D01.4c) Determine the value of i in degrees assuming the orbital radius to be a_{min}. **[2]**

[Για τα πιο πάνω, δείξετε στο γραφημα τις ποσότητες t_T, t_F και Δ , όπως σας υποδεικνύονται και στο σχήμα του παραδείγματος. Υπολογίστε τες, και βάση αυτών και τις υπόλοιπες ζητούμενες ποσότητες]

Introducing limb darkening

So far we have assumed the stellar disk to be uniformly bright. In reality, the observed brightness of the stellar disk is not uniform due to "limb darkening" — an optical effect where the central part of the stellar disk appears brighter than the edge, or the "limb".

[Αυτό το φαινόμενο συμβαίνει λόγω του ότι είναι πιο πυκνό ένα άστρο στο εσωτερικό παρά στις άκρες του, και λαμβάνεται υπόψιν στα αποτελέσματα.]

The limb darkening effect can be measured by the relative intensity $J(\theta) \equiv \frac{I(\theta)}{I(0)}$, where θ is the angle between the normal to the stellar surface at a point and the line joining the observer to that point, $I(\theta)$ is the observed intensity of the stellar disk at that point ($I(0)$ being the intensity at the centre of the stellar disk). For a distant observer, θ varies from $\theta = 0$ (centre of the disk) to $\theta \approx 90°$)(edge of the disk).

(D01.5) The table below gives measured $J(\theta)$ at a certain wavelength for the Sun. We shall assume that the same limb darkening profile holds for the star S.

θ	$J(\theta)$	θ	$J(\theta)$	θ	$J(\theta)$	θ	$J(\theta)$
0°	1.000	20°	0.971	40°	0.883	70°	0.595
10°	0.994	25°	0.950	50°	0.794	80°	0.475
15°	0.984	30°	0.943	60°	0.724	90°	0.312

The limb darkening profile can be modelled by a quadratic formula:

$$J(\theta) = 1 - a_1(1 - \cos\theta) - a_2(1 - \cos\theta)^2,$$

where a_1 and a_2 are two constants.

We shall estimate the unknown coefficients a_1 and a_2 from the given data by making a plot with suitable variables.

(D01.5a) Choose a pair of variables (x_1, y_1) which are suitable functions of θ and J, that you want to plot along x and y axes, respectively, to determine a_1 and a_2. Write the expressions for x_1 and y_1. **[2]**

If you need to define additional variables for additional plots, define them as (x_2, y_2), etc.

(D01.5b) Tabulate the values necessary for your plots. **[4]**

(D01.5c) Plot the newly defined variables on the given graph paper (mark your graph as "D01.5c"). **[7]**

(D01.5d) Obtain a_1 and a_2 from the plot. Uncertainties on the values are not needed. **[7]**

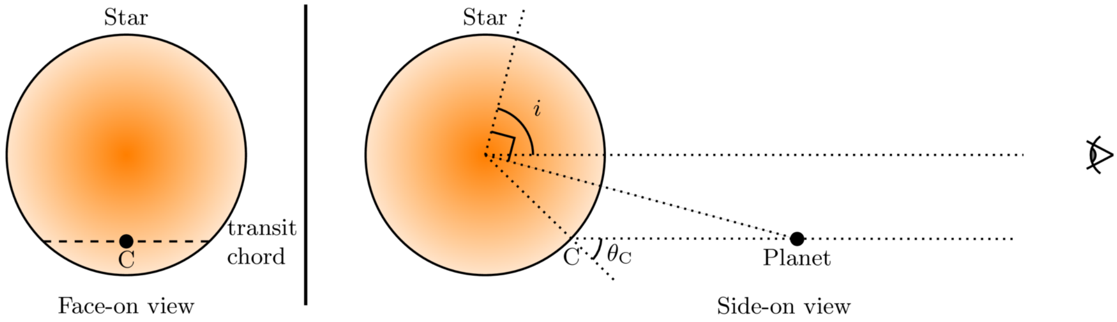
Transit in the presence of limb darkening

Now, we consider planetary transits across a limb darkened stellar disk. In the presence of limb darkening, which we shall model by the quadratic formula of $J(\theta)$ given above, the average observed intensity of the entire stellar disk (without any transit), $\langle I \rangle$, is given by:

$$\langle I \rangle = \left(1 - \frac{a_1}{3} - \frac{a_2}{6}\right) I(0)$$

Further, the dip in the light caused by the transiting planet now depends not only on the relative size of the planet and the star, $\left(\frac{R_\mathrm{p}}{R_\mathrm{s}}\right)$, but also on the intensity profile of the stellar disk along the transit chord, which in turn, depends on the angle of inclination, i.

The schematic diagram below (not drawn to scale) shows the configuration. Note that the brighter part of the star is shown in a darker shade, while the planet is shown as a black dot.



Here the relation between $\left(\frac{R_p}{R_s}\right)$ and the measured Δ from the light curve is

$$\Delta = \frac{I(\theta_C)}{\langle I \rangle}\left(\frac{R_p}{R_s}\right)^2,$$

where $I(\theta_C)$ is the intensity of the stellar disk at the midpoint of the transit chord (point C in the figure above), θ_C being the angle between the line of sight and the normal to the surface at that point. From the above it is obvious that for a given star, the same value of Δ can be produced by many combinations of the planet size, R_p, and the inclination angle i.

(D01.6) It is possible to uniquely determine both R_p and i by using data from transit lightcurves at two wavelengths, say, λ_B (blue) and λ_R (red). The limb darkening coefficients for these two wavelengths are given below:

Wavelength	a_1	a_2
λ_B	0.82	0.05
λ_R	0.24	0.20

(D01.6a) Choose the correct statement among the following that describes the relation between the maximum depth of the transit Δ for λ_B and the inclination angle (i) of the orbit and tick it (✔) in the Summary Answersheet. **[2]**

A. Δ increases with decreasing i.
B. Δ decreases with decreasing i.
C. Δ is independent of i.

(D01.6b) The maximum depth of the transit (Δ) for the "SP system" was measured to be 0.0182 and 0.0159 for λ_B and λ_R, respectively. **[4]**

Draw schematic transit light curves for both λ_B and λ_R on the given grid and label the curves by "B" and "R", respectively. Assume that the total transit duration is same for both wavelengths. The curves need not be to scale, but should represent the shapes of the light curves correctly.

[Σχεδιάστε τις 2 καμπύλες φωτός για το κάθε μήκος κύματος. Ο χρόνος διάβασης θα είναι ο ίδιος αλλά θα έχουν ποιοτικές διάφορες στο σχήμα/μορφή της καμπύλης.]

(D01.7) We shall use a graphical method to find the values of R_p and i for the SP system using the measurements of Δ at λ_B and λ_R.

(D01.7a) Write an appropriate expression connecting the relevant variables that are to be plotted. (Hint: You may consider i or b, and R_p, among the relevant variables.) **[6]**

(D01.7b) Tabulate the appropriate quantities that are to be plotted. **[5]**

(D01.7c) Draw a suitable graph and mark it as "D01.7c". **[7]**

(D01.7d) Estimate the values of R_p (in $\mathrm{R}_\odot$) and i (in degrees) from the graph. **[4]**

(D01.8) Based on the results obtained in this problem, indicate whether the planet P is “ROCKY” or “GASEOUS” by ticking (✓) the appropriate box in the Summary Answersheet. **[2]**

(D02) Predicting arrival times of coronal mass ejections on Earth **[60 marks]**

The Sun occasionally releases magnetized plasma, termed coronal mass ejections (CMEs), that originate from the surface of the Sun and propagate outwards. Accurate prediction of their arrival times at Earth is crucial for understanding and mitigating their potential effects on satellites orbiting the Earth. In this problem, we aim to predict the arrival times of CMEs by developing an empirical model, using the data of 10 CMEs.

Throughout this problem, the distance between the Sun's surface and Earth is taken to be $214 R_\odot$.

Further, assume that the Sun is not rotating. Due to electromagnetic, gravitational and drag forces, CMEs experience a variable acceleration throughout their propagation. In the first two parts of this problem , we assume that the region between the Sun and the Earth is vacuum

[Θα υπολογίσετε τον χρόνο άφιξης μιας ηλιακής καταιγίδας στην Γη, με την παραδοχή ότι ο Ήλιος δεν περιστρέφετε άρα το πλάσμα έρχεται ακτινικά προς στην Γη. Επίσης, λόγω διάφορων μεταβλητών το πλάσμα μπορεί να χάσει ή και να κερδίσει από την αρχική του ταχύτητα. Θα δείτε διάφορες περιπτώσεις και μοντέλα που εκτιμούν την διάδοση του προς την Γη.]

CMEs through vacuum.

(D02.1) The initial velocity, u, at the solar surface ($= 1R_\odot$), the final velocity, v, upon reaching Earth, and the time to arrive at Earth after leaving the surface of the Sun (in hours), τ, are given for 10 CMEs in the following table.

CME	u	v	τ
Name	(km s^{-1})	(km s^{-1})	(h)
CME-A	804	470	74.5
CME-B	247	360	127.5
CME-C	523	396	103.5
CME-D	830	415	71.0
CME-E	665	400	104.5
CME-F	347	350	101.5
CME-G	446	375	99.5
CME-H	155	360	97.0
CME-I	1016	515	67.0
CME-J	683	410	54.0

(D02.1a) Calculate the average acceleration, a, for each CME in m s^{-2}. **[3]**

(D02.1b) We assume an empirical model for the acceleration, a_{model}, of a CME, which depends on its initial velocity u as, $a_{\text{model}} = m\left(\frac{u}{u_0}\right) + \alpha$; where, a_{model} is expressed in m s^{-2}, u is expressed in km s^{-1} and $u_0 = 1.00 \times 10^3$ km s^{-1}.

Determine the constants m and α and their associated uncertainties using an appropriate graph (mark your graph as "D02.1b"). **[15]**

(D02.1c) For each CME, tabulate a_{model} in m s^{-2}. Hence calculate the root-mean-square (rms) deviation of accelerations, δa_{rms}, between the calculated acceleration, a, and the model values, a_{model}. **[4]**

[Για τα 10 CME που σας δοθήκαν, υπολογίστε την μέση επιτάχυνση το κάθε ενός. Βάλτε τα στο γράφημα σας και από αυτό υπολογίστε την τιμή των σταθερών m και α. Τώρα μπορείτε να υπολογίστε και το a_{model} για το κάθε ένα. Τέλος υπολογίστε την απόκλιση rms των τιμών μεταξύ μοντέλου και παρατήρησης.]

(D02.2) We consider two other CMEs: CME-1 and CME-2, with initial velocities, u =1044 km s^{-1} and 273 km s^{-1}, respectively.

[2 διαφορικά CMEs, με γνωστές αρχικές ταχύτητες.]

(D02.2a) Using the empirical model obtained in (D02.1b), calculate the predicted arrival times at Earth, $\tau_{1,\,\text{m}}$ and $\tau_{2,\,\text{m}}$ (in hours), for CME-1 and CME-2, respectively. **[4]**

[Χρησιμοποιήστε το προηγούμενο μοντέλο για να υπολογίσετε τον χρόνο άφιξης αυτών των 2.]

(D02.2b) The observed arrival times at Earth of CME-1 and CME-2 are 46.0 h and 74.5 h, respectively. The empirical model is considered to be VALID for a particular CME if its predicted arrival time is within 20% of its observed arrival time; otherwise, it is NOT VALID. Indicate the validity of the model for each CME by ticking (✓) the appropriate box in the Summary Answersheet. **[2]**

[Σας δίνονται οι πραγματικοί χρόνοι άφιξης. Εάν η διάφορα είναι μέχρι 20%, τότε το μοντέλο ισχύει]

CMEs in presence of solar wind

In reality, the space between the Sun and the Earth is permeated with the solar wind, which exerts a drag force on CMEs. This drag force can either decelerate or accelerate a CME, depending on the CME's velocity relative to that of the solar wind. To account for the solar wind's influence, we will use a "drag-only" model for distances $R_{\text{obs}}(t) \geq R_0$, where R_0 is the distance beyond which the drag force becomes the dominant force affecting the CME's motion.

[Σε αυτό το μοντέλο θεωρούμε ότι υπάρχει ένα σημείο R_0 , όπου από εκεί και μετά επιδρά στην διάδοση, και μοντελοποιούμε βάση αυτού του σημείου και έπειτα.]

The distance from the surface of the Sun as determined from the "drag-only" model, $R_{\text{D}}(t)$, and velocity, $V_{\text{D}}(t)$, of a CME in this model is given by

$$R_{\text{D}}(t) = \frac{S}{\gamma} \ln\left[1 + S\gamma(V_0 - V_{\text{s}})(t - t_0)\right] + V_{\text{s}}(t - t_0) + R_0$$

$$V_{\text{D}}(t) = \frac{V_0 - V_{\text{s}}}{1 + S\gamma(V_0 - V_{\text{s}})(t - t_0)} + V_{\text{s}}$$

where, $\gamma = 2 \times 10^{-8}$ km^{-1}, V_{s} is the constant speed of the solar wind, R_0 and V_0 are the distance and velocity, respectively, at time t_0, and S is the sign factor. $S = 1$ if $V_0 > V_{\text{s}}$; $S = -1$ if $V_0 \leq V_{\text{s}}$.

(D02.3) The tables below show the observed radial distance from the surface of the Sun, $R_{\text{obs}}(t)$ (measured in $R_\odot$) , as a function of time , t (in hours), for two CMEs: CME-3 and CME-4. The last data point in each table (D5 and P8, respectively) corresponds to the arrival time of the respective CME at Earth. For this part, assume $V_{\text{s}} = 330$ km s^{-1}.

[Πιό κάτω σας δίνονται μετρήσεις χρόνου και απόστασης για 2 διαφορετικά CME. Η τελευταία μετρηση αντιστοιχεί στην άφιξη των CME στη Γη. Θεωρήστε $V_{\text{s}} = 330$ km s−1]

CME-3		
Data point	t (in h)	$R_{\text{obs}}(t)$ (in $R_\odot$)
D1	0.200	6.36
D2	0.480	7.99
D3	1.22	11.99
D4	1.49	13.51
D5	58.05	214

CME-4		
Data point	t (in h)	$R_{\text{obs}}(t)$ (in $R_\odot$)
P1	1.00	4.00
P2	3.00	6.00
P3	4.00	9.00
P4	5.00	11.0
P5	21.0	43.0
P6	50.0	100
P7	85.0	170
P8	111	214

We shall evaluate if the "drag-only" model satisfactorily predicts the arrival times of these CMEs. To use this model an appropriate choice of t_0, and corresponding R_0 and V_0 needs to be made.

(D02.3a) For CME-3, take the following two cases: **[6]**
(C1) t_0 is taken as the midpoint of the interval D1 – D2
(C2) t_0 is taken as the midpoint of the interval D3 – D4
Assume the velocity remains constant in each specific interval D1–D2 and D3–D4, but may differ between the two intervals.

Using t_0, R_0, and V_0, calculate the difference between the observed and the predicted radial distance $\delta R_{\text{D}} \equiv R_{\text{obs}}(t) - R_{\text{D}}(t)$ in units of $R_\odot$ at $t = 58.05$ h, for each of the two cases.

[Πρώτα για το CME-3: θα δοκιμάσετε 2 μοντέλα. Ένα με t_0 στο μέσο των μετρήσεων D1 – D2, και άλλο με t_0 στο μέσο των μετρήσεων D3 – D4. Βρείτε τα αντίστοιχα t_0, R_0, and V_0 για την κάθε περίπτωση. Βρείτε την απόσταση που προβλέπεται από το μοντέλο να διήνυσαν για τον χρόνο που καταφθάνουν στη Γη. Υπολογίστε και την διαφορά αυτή μεταξύ παρατήρησης και μοντέλου]

(D02.3b) Evaluate $R_{\text{D}}(t)$ at points, P5, P6, P7, and P8 between the Sun and the Earth for CME-4 for the following two cases adopting the procedure similar to (D02.3a): **[4]**

(C3) t_0 is taken as the midpoint of the interval P1 – P2
(C4) t_0 is taken as the midpoint of the interval P3 – P4.

[Για το CME-4: Υπολογίστε την προβλεπόμενη απόσταση που θα διήνυσαν για τους χρόνους των P5, P6, P7, και P8. Πάλι 2 περιπτώσεις με διαφορετικά t_0]

(D02.3c) Plot $R_{\text{D}}(t)$ (in $R_\odot$) vs t (in hours) for the two cases, C3 and C4, for CME-4 at points, P5, P6, P7, and P8 (mark your graph as "D02.3c"). On the same graph, draw smooth curves of $R_{\text{D}}(t)$ for the above mentioned two cases. For this part, take the range of x axis from 0 to 180 hr. **[10]**

[Κάντε στην ίδια γραφική τις 2 περιπτώσεις του CME-4 και περάστε τις καλύτερες ευθείες από τα σημεία]

(D02.3d) Using the graph, estimate the absolute difference, $|\delta\tau|$ between the actual arrival time of CME-4 at the Earth and its time of arrival predicted by the drag-only model, for each of the cases C3 and C4. **[4]**

[Απο την γραφική, βρείτε τον χρόνο που προβλέπει το μοντέλο αυτό για να διανύσει την απόσταση από τον Ήλιο στην Γη. Βρείτε και την διάφορα τους σε σχέση με τον καταμετρημένο χρόνο]

(D02.3e) Indicate whether the following statement is TRUE or FALSE by ticking (✓) the appropriate box in the Summary Answersheet (no written justification needed): **[1]**
"The drag forces exerted by the solar wind on CMEs become dominant for CME-3 at an earlier time compared to CME-4".

(D02.4) Consider drag as the dominant force acting on 10 CMEs in part D02.1. Assume that the "drag-only" model is applicable from the surface of the Sun ($R_0 = 1\ \mathrm{R}_\odot$) and beyond, for all CMEs. Estimate and tabulate the solar wind speed V_s in km s^{-1} for each CME. Further, estimate the average solar wind speed $V_{\mathrm{s,\,avg}}$ for all 10 CMEs. **[7]**

[Θεωρήστε αυτό το μοντέλο να ισχύει και για τα αρχικά 10 CMEs. Χρησιμοποιήστε την σχέση του V_s από το προηγούμενο ερώτημα, και θυμηθείτε πως συμπεριφέρεται το πρόσημο του S. Καταγράψετε τα στον πίνακα και υπολογίστε την μέση τιμή]