(OP01) Hypothesized Planet in the Solar System [25 marks]

For this question use sky map Map-OP01. This is an angle preserving projection of the sky.

The sky projected on the planetarium dome is as seen from the solar system barycentre. The Earth's orbit as seen from this position has been marked in green both on the projected sky and in sky map Map-OP01. The first 3 minutes are for dark adaptation and familiarization of the sky.

After first 3 minutes, four Trans-Neptunian Objects (TNOs) labelled O1, O2, O3 and O4 will begin to appear on the dome. The brightness of the TNOs has been enhanced to make them visible. No other solar system object is visible in the projected sky. Time has been sped up to make their movements noticeable.

We will assume that the orbital plane of a hypothesized new planet has an inclination equal to the mean of the inclinations of the orbits of these 4 TNOs.

(OP01.1) Arrange the 4 TNOs in ascending order of the lengths of their semi-major axes and write their names in the appropriate boxes in the Summary Answersheet. Show measurements in the working sheet to justify your answer. **8**

Solution:

The orbits of the 4 TNOs are elliptical, which can be noticed from careful observations of their speed. Hence, to estimate lengths of their semi-major axes it is essential to measure the time period of one complete revolution of each TNO. The time periods of the planets are

O1: 40 sec, O2: 30 sec, O3: 48 sec, O4: 34 sec

Hence, the correct ascending order is: O2 $<$ O4 $<$ O1 $<$ O3.

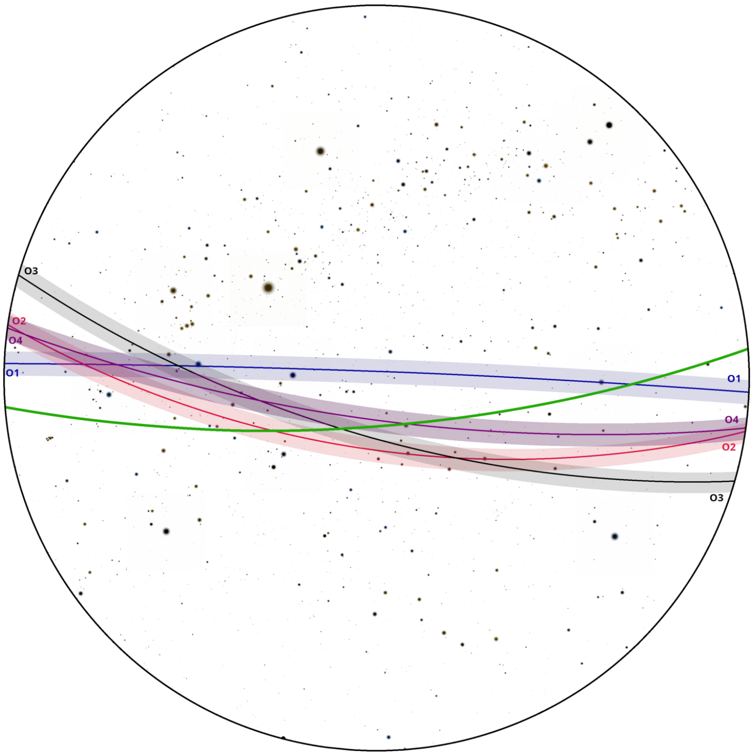
Credit of 1.0 for every pairwise correct order, and 2.0 extra for fully correct order.

O1 < O2 < O3 < O4	3.0	O2 < O1 < O3 < O4	4.0
O1 < O2 < O4 < O3	4.0	O2 < O1 < O4 < O3	5.0
O1 < O3 < O2 < O4	2.0	O2 < O3 < O1 < O4	3.0
O1 < O3 < O4 < O2	1.0	O2 < O3 < O4 < O1	4.0
O1 < O4 < O2 < O3	3.0	O2 < O4 < O1 < O3	8.0
O1 < O4 < O3 < O2	2.0	O2 < O4 < O3 < O1	5.0
O3 < O1 < O2 < O4	1.0	O4 < O1 < O2 < O3	4.0
O3 < O2 < O4 < O1	3.0	O4 < O3 < O1 < O2	2.0
O3 < O1 < O4 < O2	0.0	O4 < O1 < O3 < O2	3.0
O3 < O2 < O1 < O4	2.0	O4 < O2 < O1 < O3	5.0
O3 < O4 < O1 < O2	1.0	O4 < O2 < O3 < O1	4.0
O3 < O4 < O2 < O1	2.0	O4 < O3 < O2 < O1	3.0

(OP01.2) Trace at least 50% of the visible trajectories for each of the 4 TNOs on the map Map-OP01. From these traces find the inclination angles, i_1, i_2, i_3, i_4, of the 4 TNOs, respectively, with respect to the Ecliptic. Hence obtain the best estimate of the inclination, i_{hp}, of the plane of the orbit of the hypothesized planet. **17**

Solution:

The orbits of the TNOs can be drawn by marking a few important constellations and noticing the fact that some prominent or important stars lie along the TNOs' paths. Also the intersection points of each TNO trajectory with respect to the Earth's orbit (i.e. the Ecliptic) lie near prominent stars.



Since this map has an angle preserving (Stereoscopic) projection, measuring the angles directly using a protractor at the intersection points gives the inclination of the orbits with respect to the Ecliptic.

The measured values of inclination are:

- $i_1 = 18^\circ$
- $i_2 = 14^\circ$
- $i_3 = 20^\circ$
- $i_4 = 12^\circ$

The average inclination $\boxed{i_{\text{hp}} = 16^\circ}$.

For each TNO:

If more than 50% of the visible orbit is traced at all	0.5

If more than 50% of the traced orbit lies within the tolerance band	1.5
Between 25% to 50%	1.0

Marks for the correctness of trace will be given only if 50% of the visible orbit is traced at all.

Measurement of the inclination angles within $\pm 2^\circ$	2.0
Measurement of the inclination angles within $\pm 4^\circ$	1.0

Marks for angle will be given only if more than 25% of the traced orbit lies inside the tolerance band.

Averaging of the angles	1.0

(OP02) Observer on a Ringed Planet **[25 marks]**

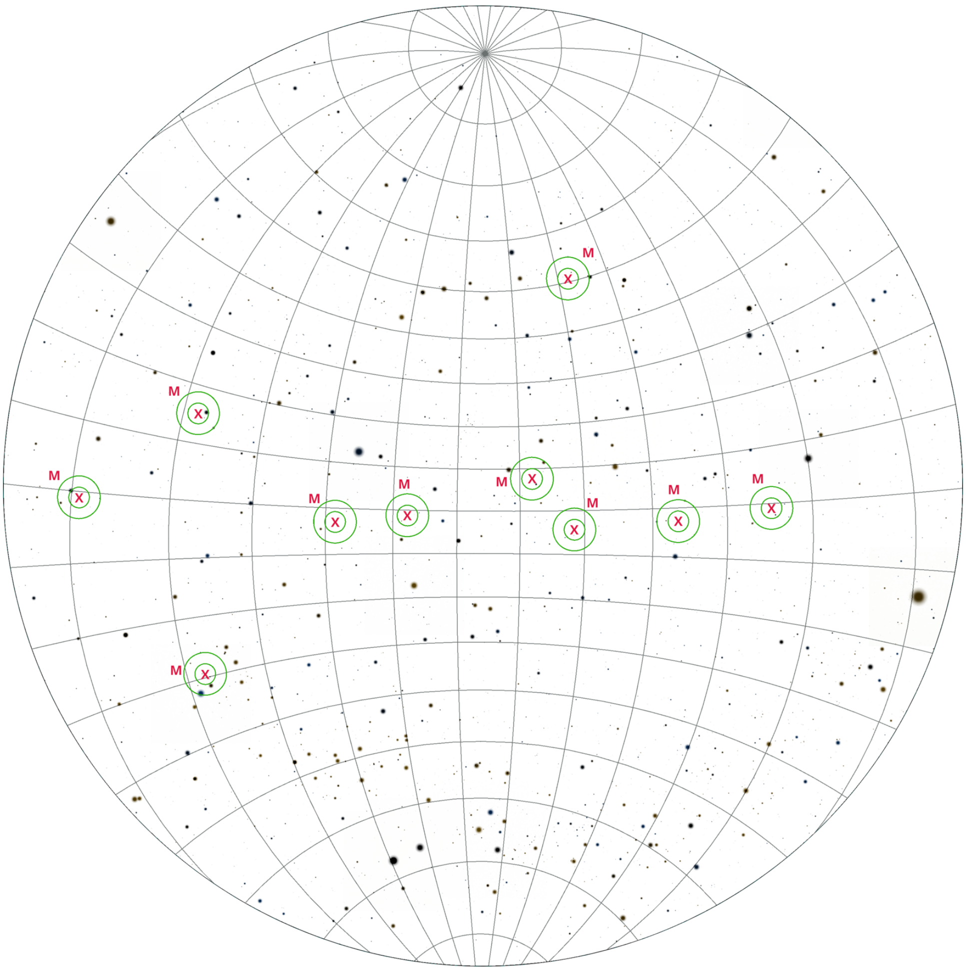
For this question use sky map Map-OP02.
The sky projected on the planetarium dome is as seen by an observer on some hypothetical ringed planet. The observer is located very near to the equator of the planet.

Assume that the ring lies in the equatorial plane of the planet and the distance of the planet from its parent star is more than 15 au. Note that except for the 10 moons of this planet no other object from this planetary system (other planets, their satellites, asteroids and even the parent star) are visible on the projected sky.

At the start of the question the sky will make 3 full rotations at the rate of 1 rotation every minute. After that the sky will stop rotating at the observer's midnight. This sky view will remain on the dome for next 7 minutes.
Solve the following questions once the sky stops rotating.

(OP02.1) Mark the positions of any 4 (only 4) out of the 10 visible moons of the planet with a ⊗ sign and label them as "M" in the sky map. **8**

Solution:

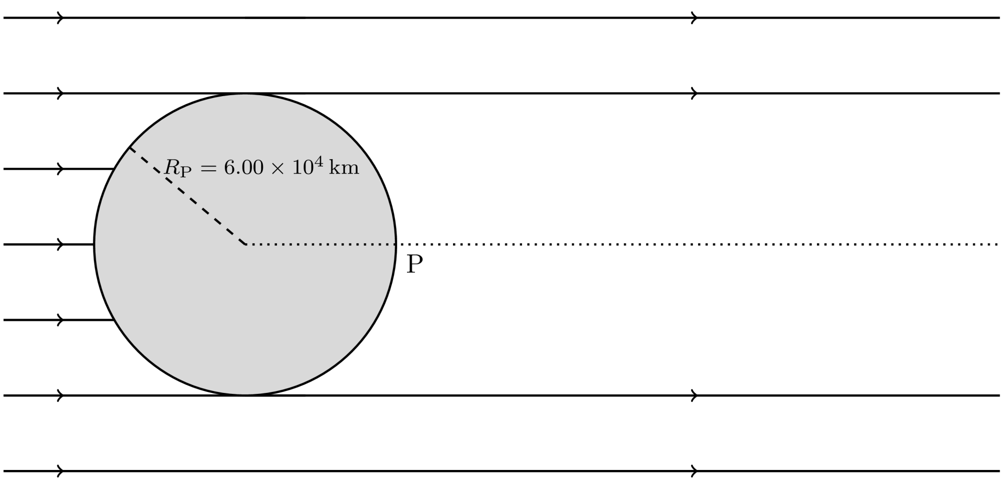


For each moon

Centre of the ⊗ within the inner circle	2.0
Centre of the ⊗ between the inner and outer circles	1.0

Use 4 correctly placed moons for marking. Every incorrectly placed moon beyond 4 gets -1.0, upto a minimum of 0.

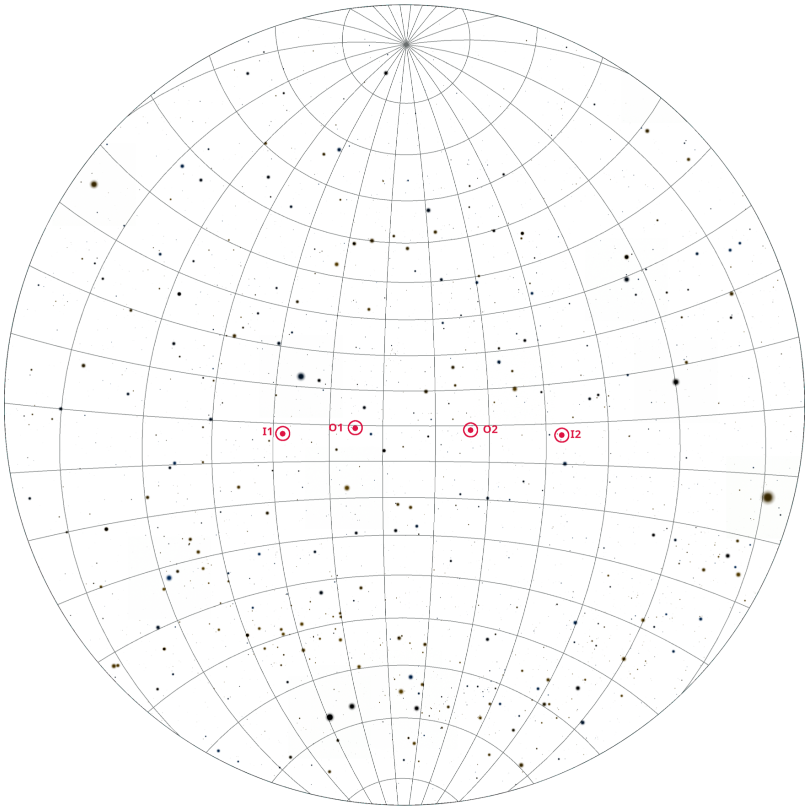
(OP02.2) The figure shown below is as seen from the top of the north pole of the planet (grey circle) and the horizontal lines are parallel light rays from its parent star. **12**



Using the view of the ring on the dome, draw the inner and outer boundaries of the ring on this diagram, reproduced in the Summary Answersheet. Your sketch must be to scale.

Solution:

The different angular extents of the shadow on the inner and outer edges of the ring are visible on the dome. These edges may be marked on the sky map (using convenient positions of the stars nearby).

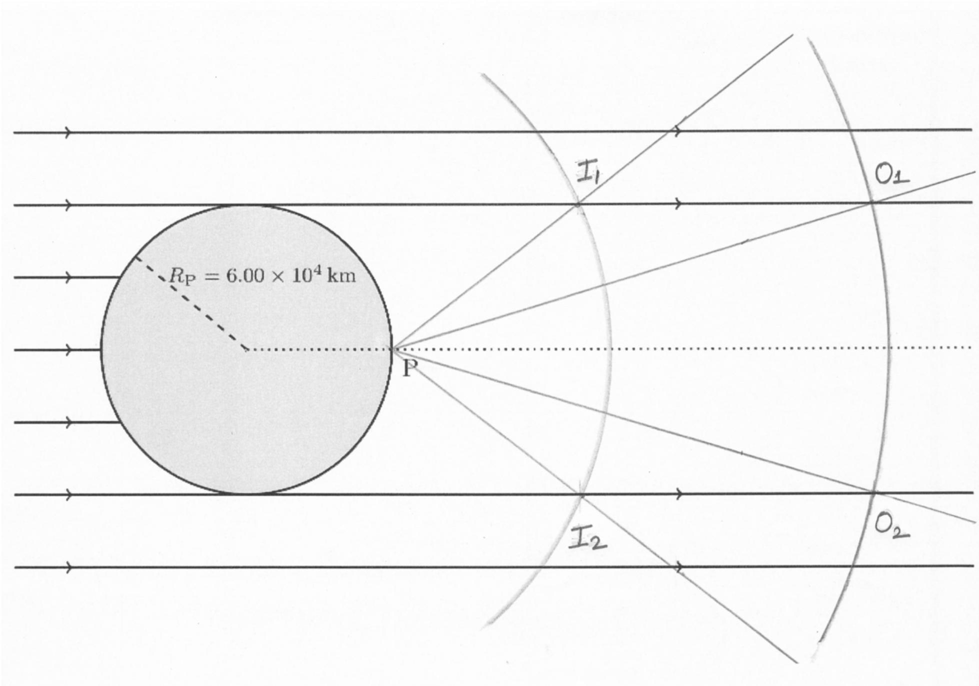


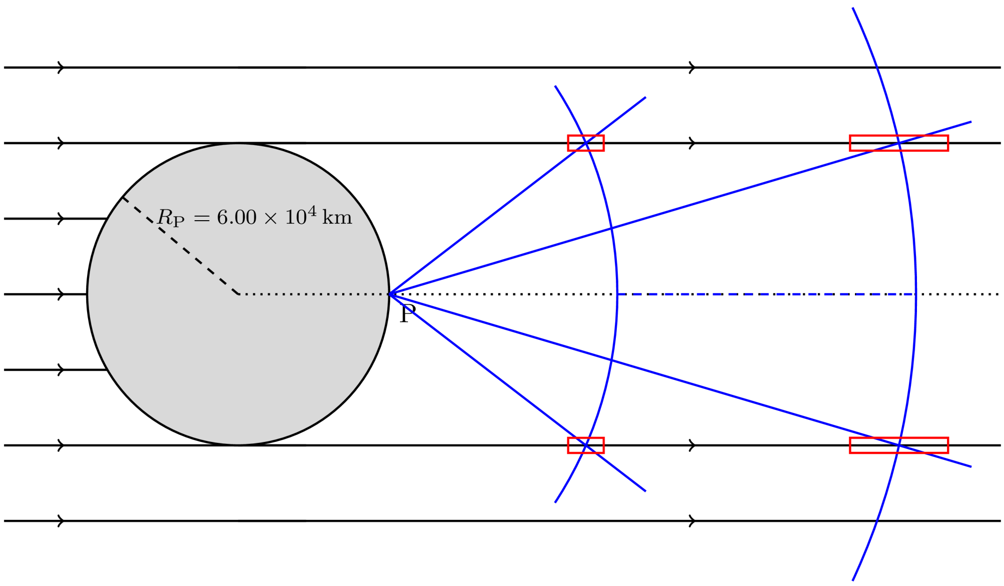
From the grid of the RA lines on the map the angular sizes are measured to be:
Inner edge: $\approx 75°$
Outer edge: $\approx 33°$

These angles need to be used in the schematic diagram to locate the rings.

Place a protractor with its baseline along the dotted line, and with its origin at P (the position of the observer).
Find the line of sight to the inner edge that subtends an angle of $75°/2 = 37.5°$ at P.
Extend this line to intersect the tangent ray.
This point marks one end of the shadow on the inner ring.
Similarly, find the other ends of the shadow on the inner edge.
Draw a circular arc (with centre at the centre of the planet) passing through these two points of intersection to show the inner edge of the ring.

Repeat for the outer edge.





Marking of each of the four intersection points of the edges with light ray within the box	2.5
Circular arcs of edges drawn	2.0

(OP02.3) Use the diagram above to determine the width of the ring, w_{ring} (in km), given that the radius of the planet is 6.00×10^4 km. **5**

Solution:

Once the edges of the ring are drawn, the width can be calculated by measuring the difference of their distances from the centre of the planet. Use a ruler to measure these distances. Also measure the radius of the planet on the diagram with the ruler to find the calibration.

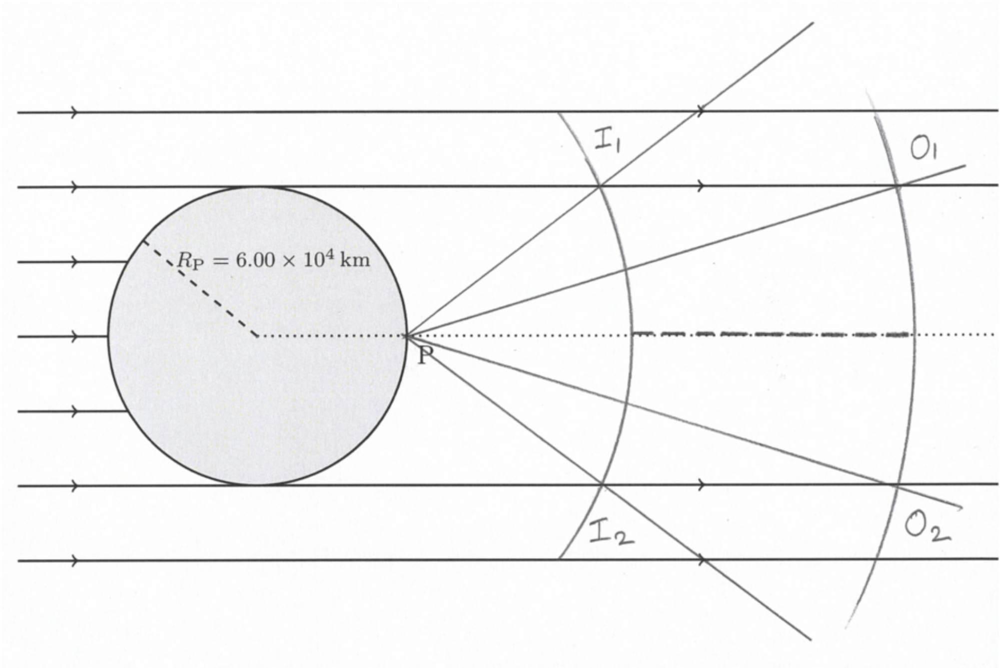
We find,

Distance between the two rings along the dotted line: 3.8 cm

Radius of the planet on the diagram: 2 cm equivalent to 6.00×10^4 km (given)

Therefore,

Width of the ring in km: $= \boxed{\dfrac{3.8\,\text{cm}}{2\,\text{cm}} \times 6.00 \times 10^4\,\text{km} = 11.4 \times 10^4\,\text{km}}$



Tolerance on values of radii of inner and outer edges

Half credit lower limit	Full credit range	Half credit upper limit
9.45×10^4 km	10.60×10^4 km to 13.40×10^4 km	14.55×10^4 km