(G01) Uncovering Galactic dark matter using a radio telescope [150 marks]

(G01.1) We estimate the resolution of our radio telescope and compare it with the angular accuracy of its pointing devices.

(G01.1a) Measure the aperture dimensions of the radio telescope (longer length a, shorter length b) and express in metres. From this, estimate the resolution corresponding to each of the dimensions separately ($\theta^{\mathrm{a}}_{\mathrm{res}}$ and $\theta^{\mathrm{b}}_{\mathrm{res}}$, respectively) in degrees. Tick (✓) the dimension which gives you the higher resolution. **7**

Solution:

1. Given Parameters

- **Frequency** (f): 1420.40575 MHz or 1.42040575×10^9 Hz
- **Horn Dimensions**:
  - (a): 56.5 cm or 0.565 m
  - (b): 41.5 cm or 0.415 m
- **Speed of light** (c): $299,792,458$ m/s

2. Calculate the Wavelength (λ)

$$\lambda = \frac{c}{f}$$

$$\lambda = \frac{299,792,458 \text{ m/s}}{1,420,405,750 \text{ Hz}} \approx \mathbf{0.2111} \text{ meters}$$

3. Calculate the Angular Resolution (HPBW)

Resolution along Larger Dimension (θ_a)

This resolution is associated with the larger dimension, a.

$$\theta_a \approx \frac{180}{\pi}\left(\frac{\lambda}{a}\right)$$

$$\theta_a \approx \frac{180}{\pi}\left(\frac{0.2111 \text{ m}}{0.565 \text{ m}}\right) \approx \mathbf{21.4^\circ}$$

Resolution along Smaller dimension (θ_b)

This resolution is associated with the smaller dimension, b.

$$\theta_b \approx \frac{180}{\pi}\left(\frac{\lambda}{b}\right)$$

$$\theta_b \approx \frac{180}{3.14} \times \left(\frac{0.2111 \text{ m}}{0.415 \text{ m}}\right) \approx \mathbf{29.2^\circ}$$

Measurement in two directions (1+1)	2.0
Estimating wavelength	1.0
Writing the formula for resolution	1.0
Computing the correct value in both directions (1 + 1)	2.0
Choosing the dimension which gives the smaller of the two angles	1.0

(G01.1b) In order to compare with the above resolution, write down the least count **2** (smallest measurable unit) of the protractor (θ^{az}_{min}) and the digital inclinometer (θ^{alt}_{min}).

Solution:
$\theta^{alt}_{min} = 0.01°$ and $\theta^{az}_{min} = 1°$

(G01.2) Open the telescope operating software and verify using Tab 1 that the system is working. For each longitude, **25**

- Use Tab 2 to enter a Galactic coordinate (longitude ℓ, latitude $b = 0$) and note the time, corresponding altitude, azimuth and v_{corr} in the Table in Summary Answersheet.
- Point to this altitude and azimuth and use Tab 3 to record the data 'ℓ.csv' (e.g. a file named '30.csv' could contain data corresponding to longitude 30°).

Repeat this procedure for all the different longitudes from the table.

Solution:
Date: 24th July 2025

Galactic Longitude	Azimuth (°)	Altitude (°)	V_{corr} kms^{-1}	Time
30°	119.7°	50.5°	-7.484	21:01:11
40°	104.2 °	51.3 °	-11.802	21:03:31
50°	88.9°	50.0°	-15.761	21:05:33
60°	75.0°	46.9°	-19.240	21:08:16
70°	61.8°	50.1°	-22.078	21:47:57

Azimuth for 5 longitudes		5.0
Altitude for 5 longitudes		5.0
v_{corr} for 5 longitudes		5.0
if sign of v_{corr} is not maintained (-0.5 for each longitude)	-2.5	
Time for 5 longitudes		5.0
Correct source profile with central HI line feature above two side peaks (1.0 × each longitude)		5.0

(G01.3) Use Tab 3 to record the calibration data 'sky.csv' and 'ground.csv' by pointing the telescope towards a region of the sky away from the Galactic plane, and then towards the ground, respectively. Add the altitude and azimuth you pointed to and the time when you carried out each of these measurements in the Summary Answersheet. **14**

Solution:

Altitude for sky away from galactic plane	2.0
Azimuth for sky away from galactic plane	2.0
Time for sky	1.0
Time for ground	1.0
Quality of ground data	3.0
Quality of sky data	3.0

Difference between ground and sky data is > 3dB	2.0

(G01.4) Perform the gain and temperature calibration using the procedure corresponding to Tab 4. **7**

Solution:

Region chosen for masking the contaminating line	3.0
Value of T_{sys}	2.0
Correct gain	2.0

(G01.5) Now for each observation 'ℓ.csv', use Tab 5 to obtain the temperature spectrum at the 5 different longitudes ℓ.

(G01.5a) Determine the most redshifted frequency f_{obs} of the HI line (which has a temperature of 5 K above the baseline) corresponding to a given Galactic longitude (ℓ), and tabulate in the Summary Answersheet. **19**

Solution:

Galactic Longitude	f_{obs} [MHz]
30°	1420.0207
40°	1420.0288
50°	1420.1096
60°	1420.1503
70°	1420.2479

Value of f_{obs} at 5K for 5 longitudes	10.0
Bonus for completing all 5 measurements	4.0
Baseline coinciding with 0 in the line temperature curve for 5 longitudes	5.0

(G01.5b) Calculate $v^{\text{obs}}_{\text{Earth}}(\ell)$ and $v^{\text{max}}_{\text{LSR}}(\ell)$ and tabulate in the Summary Answersheet. **10**

Solution:

Galactic Longitude	$v^{\text{obs}}_{\text{Earth}}(\ell)$ [kms^{-1}]	$v^{\text{max}}_{\text{LSR}}(\ell)$ [kms^{-1}]
30°	81.325	73.841
40°	79.615	67.813
50°	62.549	46.788
60°	53.953	34.713
70°	33.339	11.261

Value of $v^{\text{obs}}_{\text{Earth}}(\ell)$ for 5 longitudes	5.0
Value of $v^{\text{max}}_{\text{LSR}}(\ell)$ for 5 longitudes	5.0

(G01.5c) Using values of v_{LSR}^{max} at each of the observed Galactic longitudes, calculate the rotation velocity $v_{rot}(R)$ and the Galactic radius for the maximum redshifted emission for each of the 5 Galactic longitudes. Tabulate all these values in the Summary answersheet. **10**

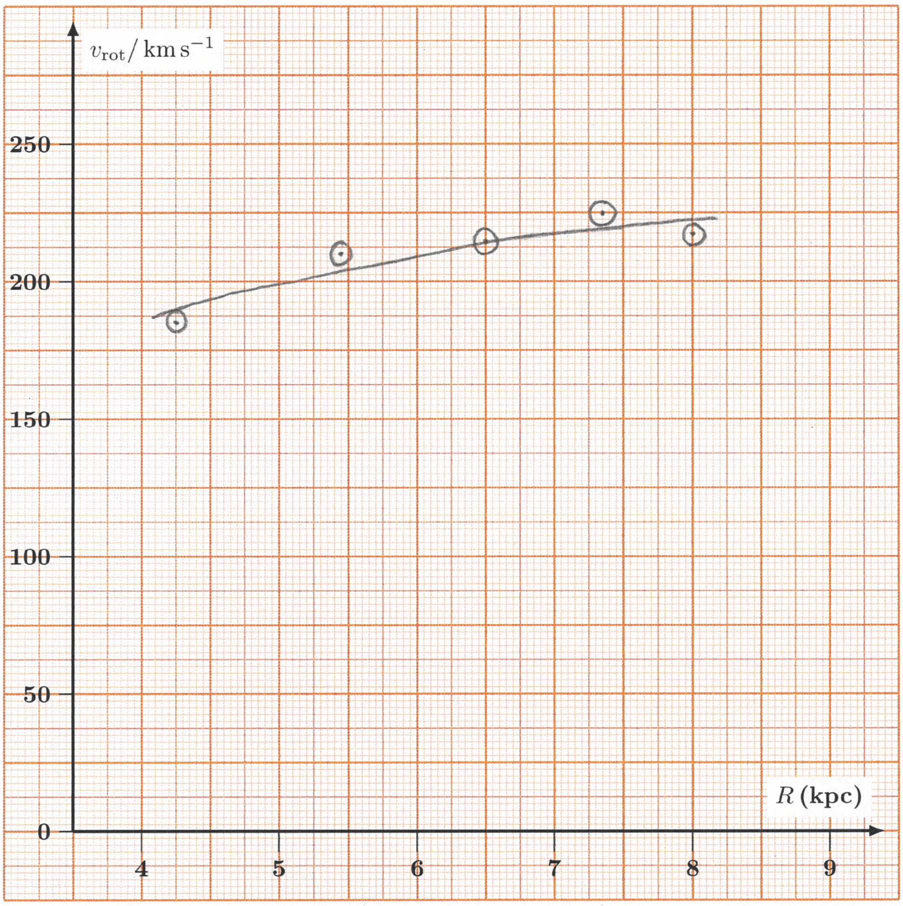
Solution:

Galactic Longitude	$v_{rot}(R)$ $/10^3 ms^{-1}$	Galacto-centric Radius, R kpc
30°	183.791	4.25
40°	209.166	5.46
50°	215.255	6.51
60°	225.180	7.36
70°	217.947	7.99

Value of v_{rot} for 5 longitudes	5.0
Value of R for 5 longitudes	5.0

(G01.6) Plot the rotation velocity versus Galacto-centric radius on the graph-sheet provided as a part of Summary Answersheet and draw a smooth curve going through these points. **6**

Solution:



Plotting of points		4.0
	all 5 points correctly plotted	4.0
	4 points correctly plotted	3.0
	3 points correctly plotted	2.0
	2 points correctly plotted	1.0
	less than 2 points plotted	0.0
Best fit/Smooth Curve		2.0

(G01.6a) Assuming a spherically symmetric mass distribution, estimate the enclosed mass within the corresponding radius of your observations using the formula: **10**

$$M_{\text{encl}}(R) = \frac{v_{\text{rot}}^2 R}{G},$$

where R is galacto-centric radius, v_{rot} is the rotation velocity, and G is the gravitational constant. Express your answer in units of solar mass.

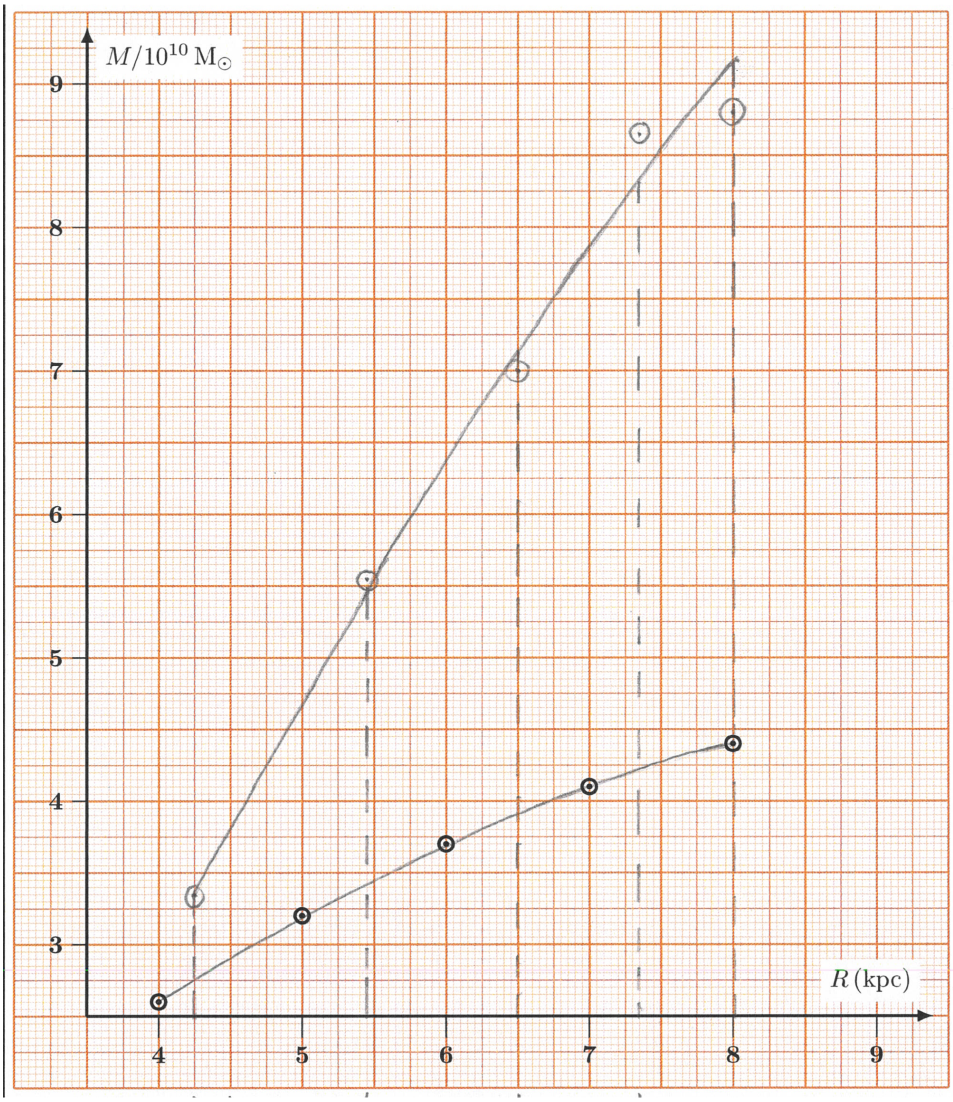
Solution:

Galactic longitude	Galacto-centric radius [kpc]	M_{encl} [$10^{10} M_\odot$
30°	4.25	3.34
40°	5.46	5.55
50°	6.51	7.01
60°	7.36	8.67
70°	7.98	8.82

For 5 longitudes	10.0

(G01.6b) The mass in ordinary baryonic matter enclosed at different Galacto-centric radii, R, of the Milky Way is shown with $\odot$ symbols in the graph given in the answer-sheet. Plot $M_{\text{encl}}(R)$ from your measurements in the same graph-sheet, and draw two smooth physically correct curves, one for each set of measurements. **12**

Solution:



all 5 points of $M_{\rm encl}$ correctly plotted	5.0
Physically correct smooth curves (monotonic 2 × 1.5, non-intersecting 2.0)	5.0
Interpolating $M^{\rm bary}$ at the location of the measurements.	2.0

(G01.6c) Calculate the value of enclosed dark matter mass, $M_{\rm dm}(R)$, and record it in the Table in the Summary answersheet. **10**

Solution:

R /kpc	$M_{\rm encl}$ / $10^{10} M_\odot$	M_{bary} /$10^{10} M_\odot$	M_{dm} /$10^{10} M_\odot$
4.0	2.8	2.6	0.2
5.0	4.7	3.2	1.5
6.0	6.6	3.7	2.9
7.0	8.1	4.1	4.0
8.0	8.6	4.4	4.2

Reading M_{encl} from the graph for given five R	5.0
Calculating M_{dm} for the given five R	5.0

(G01.7) Estimate the sensitivity of your observation per spectral bin in units of temperature in K given that the spectrum has a total of 512 bins spanning a total frequency range of 2.048 MHz. **9**

Solution:

1. Calculate Channel Bandwidth (Δf)

Determine the frequency range, or bandwidth, covered by a single "bin" or channel in the antenna.

- **Formula**: $\Delta f = \frac{\text{Total Frequency Range}}{\text{Number of Bins}}$
- **Given Data**:
  - Total Frequency Range $= 2.048$ MHz or $2,048,000$ Hz
  - Number of Bins $= 512$
- **Calculation**:

$$\Delta f = \frac{2,048,000 \text{ Hz}}{512} = \mathbf{4,000 \text{ Hz}} \text{ (or 4 kHz)}$$

2. Calculate Sensitivity (σ_T)

Next, we use the radiometer equation with the channel bandwidth, integration time and the system temperature.

- **Formula**:

$$\sigma_T = \frac{T_{sys}}{\sqrt{\Delta f \cdot t_{int}}}$$

- **Given Data**:
  - $T_{sys} = 115$ K (Measured T_{recv} + assumed T_{ant} of 5 K)
  - $\Delta f = 4,000$ Hz
  - Integration Time (τ) $= 60$ s
- **Calculation**:
  Sensitivity

$$= \sigma_T = \frac{115 \text{ K}}{\sqrt{4000 \text{ Hz} \times 60 \text{ s}}} = \frac{115 \text{ K}}{\sqrt{240,000}} \approx \mathbf{0.235 \text{ K}}$$

Determination of T_{sys} from calibration tab, using T_{recv}	4.0
Calculation of Δf	2.0
Using correct t_{int}	1.0
Calculation of RMS noise accurate to two significant digits (i.e., sensitivity) with unit	2.0

(G01.8) Which of the following parameter(s) will improve if observations were made with a horn antenna of larger aperture dimensions. Tick against the correct option(s) in the Summary answersheet. **9**

A. Sensitivity σ_T for $T_{\text{ant}} = 5\,\text{K}$
B. Angular resolution
C. Estimation of v_{rot}
D. Resolution in frequency

Solution:

Correct Options:

B. Angular resolution

C. Estimation of v_{rot}

Each correct option chosen	3.0
Each incorrect option chosen	-2.0
Bonus marks if only correct options are ticked	3.0