1. Famous astronomical events (10 p)

Arrange the following astronomy-related events in chronological order from the oldest to the most recent. **Write the correct serial number (between 1 and 11) in the appropriate box on the answer sheet.**

1. Launch of the Hubble Space Telescope
2. Viking probes arrived at planet Mars
3. Discovery of Phobos and Deimos
4. Latest perihelion of comet 1/P Halley
5. Discovery of Ceres (asteroid / dwarf planet)
6. Discovery of Uranus (planet)
7. First successful measurement of a stellar parallax
8. Discovery of the first planetary nebula
9. Discovery of stellar populations (I and II)
10. First identification of a quasar with an optical source
11. Discovery of the expansion of the Universe

2. Deflection of radio photons in the gravitational field of solar system bodies (10 p)

A. Eddington and F. Dyson from Principe, and C. Davidson and A. Crommelin from Sobral, Brazil measured the deflection of light coming from stars apparently very close to the Sun during the total solar eclipse in 1919. The deflection was found to agree with the theoretically predicted value of 1.75".

A light ray (or photon) which passes the Sun at a distance d is deflected by an angle

$$\Delta\theta \propto \frac{4GM_\odot}{dc^2}$$

The present-day accuracy of the VLBI (Very Long Baseline Interferometry) technique in the radio wavelength range is 0.1 mas (milliarcsecond). Is it possible to detect a deflection of radio photons from a quasar by (a) Jupiter, (b) the Moon? Estimate the angle of deflection in both cases and **mark "YES" or "NO" on the answer sheet**.

3. The supermassive black hole in the centre of Milky Way Galaxy and M87 (10 p)

The first image of a black hole was constructed recently by the international team of the Event Horizon Telescope (EHT). The imaged area surrounds the supermassive black hole in the centre of the galaxy M87. The observations resulting in the final image were carried out at a wavelength $\lambda = 1.3$ mm where the interstellar extinction is not prohibitively large.

a) How large an instrument would be needed to resolve the shadow (in effect the photon capture radius, which is three times the size of event horizon) of a supermassive black hole in the centre of a galaxy? Express the result as a function of the distance d and the mass M of the black hole. (6 p)

b) Give the size of the instrument in units of Earth radius for

i. the supermassive black hole in the centre of M87, (1 p)
($d_{\text{BH-M87}} = 5.5 \times 10^7$ ly, $M_{\text{BH-M87}} = 6.5 \times 10^9\ \text{M}_\odot$)

ii. and Sgr A*, the supermassive black hole of our own galaxy, Milky Way. (1 p)
($d_{\text{Sgr A*}} = 8.3$ kpc, $M_{\text{Sgr A*}} = 3.6 \times 10^6\ \text{M}_\odot$)

c) What type of technology is needed for the development of such an instrument? **Mark the letter of your answer with × on the answer sheet.** (2 p)

(A) Gravitational lensing by dark matter
(B) Interferometry with an array of radio telescopes
(C) Photon deceleration in a dense environment
(D) Reducing the effect of incoming wavefront distortions
(E) Neutrino focusing with strong electromagnetic fields

4. Improving a common reflecting telescope (10 p)

A student has an average quality Cassegrain telescope, with primary and secondary mirrors having $\varepsilon_1 = 91\ \%$ reflectivity aluminium layers.

a) What will be the change in the limiting magnitude of this telescope by replacing the mirror coatings with "premium" quality $\varepsilon_2 = 98\ \%$ reflectivity ones? (5 p)

b) Assuming the student also uses a star diagonal mirror, also with reflectivity ε_1 with the original telescope - what will be the improvement if he/she also replaces this piece with an $\varepsilon_3 = 99\ \%$ reflectivity ("dielectric" mirror) model, combined with the new ε_2 mirrors? (3 p)
(A star diagonal mirror is a flat mirror, inclined to the optical axis by 45°.)

c) Is this difference obviously detectable by the human eye? **Mark "YES" or "NO" on the answer sheet.** (2 p)

Consider the whole visual band and disregard any wavelength dependence and geometric effects.

5. Cosmic Microwave Background Oven (10 p)

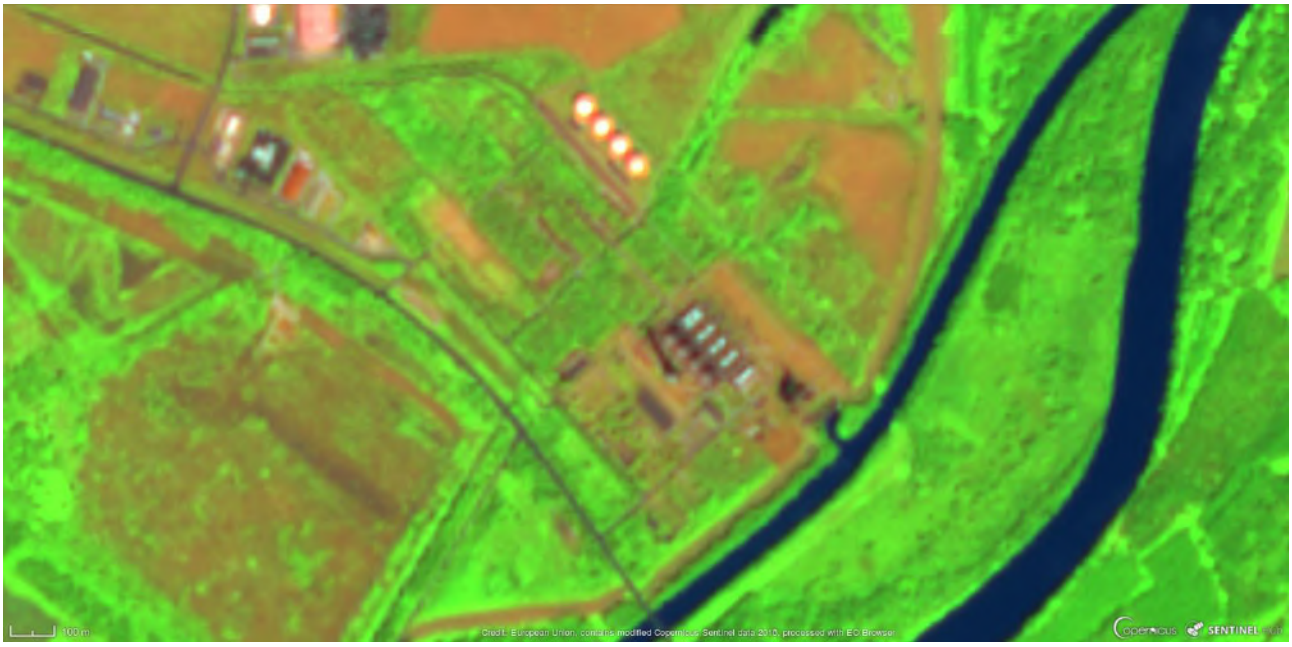
Since the human body is made mostly of water, it is very efficient at absorbing microwave photons. Assume that an astronaut's body is a perfect spherical absorber with mass of $m = 60\ \mathrm{kg}$, and its average density and heat capacity are the same as for pure water, i.e. $\rho = 1000\ \mathrm{kg\ m^{-3}}$ and $C = 4200\ \mathrm{J\ kg^{-1}\ K^{-1}}$.

a) What is the approximate rate, in watts, at which an astronaut in intergalactic space would absorb radiative energy from the Cosmic Microwave Background (CMB)? The spectral energy distribution of CMB can be approximated by blackbody radiation of temperature $T_{\mathrm{CMB}} = 2.728\ \mathrm{K}$. (5 p)
b) Approximately how many CMB photons per second would the astronaut absorb? (3 p)
c) Ignoring other energy inputs and outputs, how long would it take for the CMB to raise the astronaut's temperature by $\Delta T = 1\ \mathrm{K}$? (2 p)

6. The height of the chimney of Tiszaújváros power plant (20 p)

The European Copernicus Earth-observation programme operates two Sentinel-2 remote sensing satellites. These satellites orbit the Earth on Sun-synchronous polar orbits at about 800 km altitude. They pass over a given area once every few days, always taking images at the same local time (accurate to within a few minutes). The cameras are sensitive to 13 different optical and near-infrared spectral bands. The resolution of the images is 10 meters.

The 3rd tallest building in Hungary is the chimney of a power plant near the town Tiszaújváros. You can see two Sentinel-2 satellite images. One is from June 29, another one is from December 16, close to the summer and winter solstices, respectively. The orientation of the images is as normal, i.e. north is up and east is to the right.



The estimated shadow lengths, based on the images above and the scales given in the lower-left corners, of them are $x_1 = 125$ m and $x_2 = 780$ m. Answer the following questions:

a) On which date do we expect the shadow to be longer? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) on June 29
(B) on December 16

b) At which time of the day did the Sentinel-2 satellites fly over this area? **Mark the letter of your answer with × on the answer sheet.** (1 p)

(A) early morning
(B) late morning
(C) early afternoon
(D) late afternoon

c) Based on the given shadow lengths, estimate the height of this chimney. For this calculation only, assume that the satellite images were taken at local noon. (16 p)

d) What could affect the accuracy of the chimney's derived height (more than one choice is possible)? **Mark the letter of your answer with × on the answer sheet.** (2 p)

(A) The oblate spheroid shape of the Earth.
(B) Limited resolution of the satellite images and the ill-defined edge of the shadow.
(C) The elevation of the base of the tower above sea level.
(D) Seasonal variation in the tilt of the Earth's rotational axis.
(E) Taking into account the effect of atmospheric refraction.

7. Effect of sunspots on solar irradiance (20 p)

Since 1978, the solar constant has been almost continuously measured by detectors on-board artificial satellites. These accurate measurements revealed that there are seasonal, monthly, yearly, and longer timescale variations in the solar constant. While the seasonal variations have their origins in the periodically varying Earth–Sun distance, the decade-long quasi-cyclic variations mainly depend on the activity cycle(s) of the Sun.

a) Calculate the value of the solar constant at the top of the Earth's atmosphere, when the Earth is 1 au from a perfectly quiet Sun, assuming that Sun emits as a perfect black-body. (4 p)

b) Calculate the solar constant of this perfectly quiet Sun in early January and early July, and find their ratio. (4 p)

c) Calculate the solar constant again at 1 au in the presence of a near equatorial sunspot with mean temperature of $T_{\mathrm{sp}} = 3300$ K and diameter of sunspot, $D_{\mathrm{sp}} = 90\,000$ km. Calculate the ratio - blank Sun to Sun with sunspot. (7 p)

Assume the sunspot is circular and ignore the effects of its spherical projection. Neglect any other activity features. Also assume that the Sun is rotating fast enough, hence solar irradiance is still isotropic.

d) In reality solar irradiance is no longer isotropic. Calculate the ratio of solar irradiance for the cases when the sunspot is not visible from the Earth to the case when it is fully visible. (5 p)

8. Amplitude variation of RR Lyrae stars (20 p)

Hungarian astronomers significantly contributed to the study of pulsating variable stars of RR Lyrae type, which show cyclic amplitude modulation in their light variation (Blazhko effect)

The light variation of an RR Lyrae star is observed at two different wavelengths: $\lambda_1 = 500$ nm and $\lambda_2 = 2000$ nm. At each wavelength, we see into the star at a different depth. We refer to the depths as layer 1 and layer 2. One can approximate the light intensity of a star at a given wavelength by the radius and the black-body radiation intensity at the appropriate depth. Moreover, the Wien approximation can be used for the black-body radiation to calculate the emitted power from unit surface area:

$$F(\lambda, T) \propto \frac{1}{\lambda^5} \exp\left(-\frac{hc}{k\lambda T}\right),$$

where h is the Planck constant, k is the Boltzmann constant, and c is the speed of light. To simplify the calculations we introduce a new constant $C_b = hc/k \approx 0.0144$ m K.

a) Assume that the temperature varies between $T_1 = 6000$ K and $T_2 = 7400$ K in each of the layers and ignoring radius variation, what is the ratio of the amplitudes of variation in magnitudes at the two wavelengths? (5 p)

b) What is the peak to peak amplitude of the light curve at λ_1? Use the magnitude scale. (3 p)

c) Ignoring temperature variation, what is the contribution of the radius variation to the light-curve amplitude for a given wavelength, if $R_{\min} = 0.9\,\langle R \rangle$ and $R_{\max} = 1.05\,\langle R \rangle$? $\langle R \rangle$ is the mean radius of the given layer. (3 p)

d) Recent observations and models show that the radius of the photosphere is only minimally modulated during the Blazhko cycle; however, the temperature variation is significant. As a result, the amplitude of light curve itself keeps varying. Let us assume that during the minimum amplitude of pulsation the temperature variation is reduced to $T_{\min} = 6100$ K and $T_{\max} = 6900$ K. What is the modulation amplitude at the two wavelengths? For the maximum amplitude, use the temperature values given in part a). (5 p)

e) Which statement is correct? (There can be more than one correct selection.) **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) It is easier to observe the Blazhko effect in the infrared band.
(B) The temperature variation dominates the visual light curve.
(C) If we neglect the radius variation, the amplitude is inversely proportional to the wavelength.
(D) Multi-colour observations are not useful in understanding the Blazhko effect.

9. Distance of the Lagrangian point L_2 of the Earth–Moon system (20 p)

On 3 January 2019, the Chinese spacecraft Chang'e-4 landed on the far side of the Moon in the area of the von Kármán crater, which was named after the world famous Hungarian-born physicist Theodore von Kármán.

As the Earth remains below the horizon of the spacecraft all the time, a relay station is also necessary for the communication with mission control on the Earth. For this purpose, The Chinese Space Agency launched a spacecraft, Queqiao, which was placed into a halo orbit around the outer Lagrangian point of the Earth–Moon system, L_2, the far side of the Moon.

Calculate the distance (h) of this satellite above the surface of the Moon. The Moon's orbit should be considered as a perfect circle with a radius of $R = 384\,400$ km. Neglect perturbations from the Sun and other planets.

Hint: You can use the following approximation: $1/(1+x)^2 \approx 1 - 2x$, if $|x| \ll 1$.

10. South → East → North (20 p)

Consider the Earth as a perfect, rigid, sphere with radius $R = 6378$ km. There exist some points on the surface of the Earth from which we can travel first 6378 km South, then 6378 km East, and after that 6378 km North, and as a result return to the original point of departure. Find such points and paths. Calculate the geographic coordinates of the turning points of your solutions and draw the paths.

For the sake of simplicity measure the geographic longitude from 0° to +360° eastward from Greenwich, the geographic latitude from 0° to +90° north of the Equator and from 0° to −90° south of the Equator. Solutions resulting from rotational symmetry should not be considered to be different.

11. Identification of light curves of types of selected variable stars **(25 p)**

TESS (Transiting Exoplanet Survey Satellite) is NASA's most recent exoplanet hunter mission. It has been surveying the southern sky to locate exoplanets around the brightest and closest stars, along with a large number of time-variable phenomena (pulsating and eclipsing variable stars, supernovae, stellar flares, and asteroids among others).

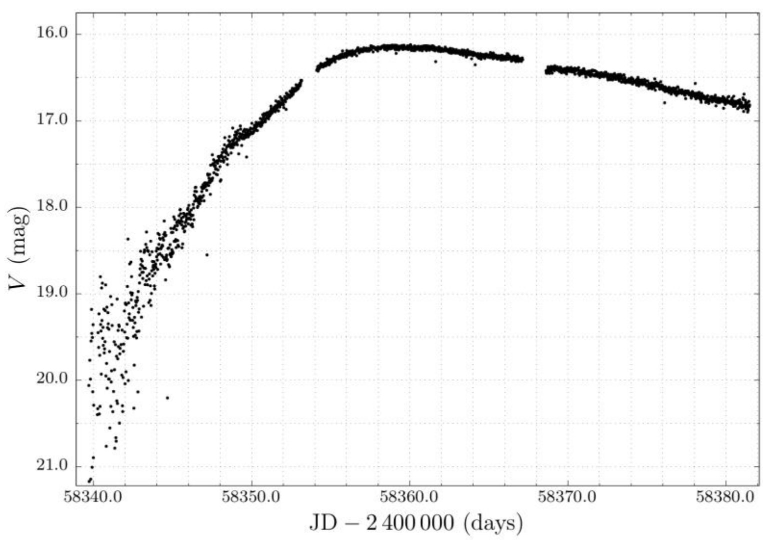
On the separate sheet you will find graphs with light curve plots of 8 periodic variable stars (intrinsic, eclipsing and rotational; namely FO Eri, RW Dor, 24 Eri, TIC 147272181, ST Pic, UY Eri, VV Ori, AH Col) from the TESS target list, numbered from 1 to 8. The horizontal axis is BJD − 2 400 000 in days, and the vertical axis is V in magnitudes, where BJD is the Barycentric Julian Date, and V is the visual brightness.

a) Below is a list of variable star types. Match each light curve of the star with its variability type **by writing its number into the rectangle on the answer sheet**. (8 p)

- Heart-beat star
- RR Lyrae type (RRab subclass) pulsating variable star
- Eclipsing binary of Algol type (semi-detached) with a pulsating component
- α^2 CVn pulsating variable star
- W Vir type (Population II) Cepheid pulsating variable star
- Detached eclipsing binary with strong reflection effect
- Contact eclipsing binary of W UMa type
- Rotationally variable (spotted) star

b) Based on the light curve plots, estimate the periods of each variable star in days. **Give your answers on the answer sheet up to 2 decimal places.** Periods in the range ±5% of the true periods are acceptable. (16 p)

c) What type of astronomical object produced the TESS light curve shown in the figure below? **Mark the letter of your answer with × on the answer sheet.** (1 p)



(A) Microlensing event
(B) Saturated galaxy due to the proximity of Mars
(C) Comet's coma close to the edge of the camera field
(D) Supernova in a distant galaxy
(E) Superflare on a supergiant star

12. Distance to a Near-Earth Asteroid (25 p)

Assume that a Near-Earth Asteroid is observed by two astronomers, one from Nagykanizsa, Hungary and one from Windhoek, Namibia. The longitudes of the two cities are exactly 17° east of Greenwich. They observe the asteroid when it crosses their respective meridians. The Nagykanizsa-observer sees the asteroid 25° south of his zenith at this instant, while the Windhoek-observer sees it 45° north of his zenith at the same instant. The latitudes of the two cities are 46°27’ N and 22°34’ S, respectively. The sites of both astronomers are at sea level.

a) Draw a diagram of the geometric configuration. (5 p)

b) What is the distance of this asteroid from the centre of the Earth, expressed in units of Earth-radii and the average Earth-Moon distance? Provide a solution which makes use of all available information. Neglect the effect of the atmospheric refraction. (20 p)

13. Distance to the Coma galaxy cluster (40 p)

The Coma galaxy cluster (Abell 1656) has an angular diameter on the sky of about 100 arcminutes, and contains more than 1000 individual galaxies, most of which are dwarf and giant ellipticals orbiting the common center of mass of the cluster in approximately circular orbits. The table below lists the measured radial velocities of a few individual cluster member galaxies.

No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)	No.	v_r (km/s)
1	6001	6	7116	11	7156	16	7111
2	7666	7	7004	12	7522	17	8292
3	6624	8	4476	13	7948	18	5358
4	5952	9	6954	14	4951	19	4957
5	5596	10	8953	15	7797	20	7183

a) Derive the distance of the cluster from the mean radial velocity of the galaxies listed in the table. (8 p)

b) Estimate the physical diameter of the cluster (in Mpc). (4 p)

c) The virial theorem states that if the galaxy cluster is in dynamic equilibrium, then the mean kinetic energy, $\langle K \rangle$, and the mean gravitational potential energy, $\langle U \rangle$, are related by

$$-2\langle K \rangle = \langle U \rangle$$

assuming the Coma cluster is spherical.

For simplicity, assume that each galaxy has approximately the same mass, m.

Use the virial theorem to prove that, in this case, the cluster mass M (also called as the *virial mass*) can be expressed as

$$M = \frac{5R}{G}\sigma_r^2$$

where σ_r^2 is the velocity dispersion of the cluster. (10 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the data in the table, estimate the virial mass of the Coma cluster in solar masses. (12 p)

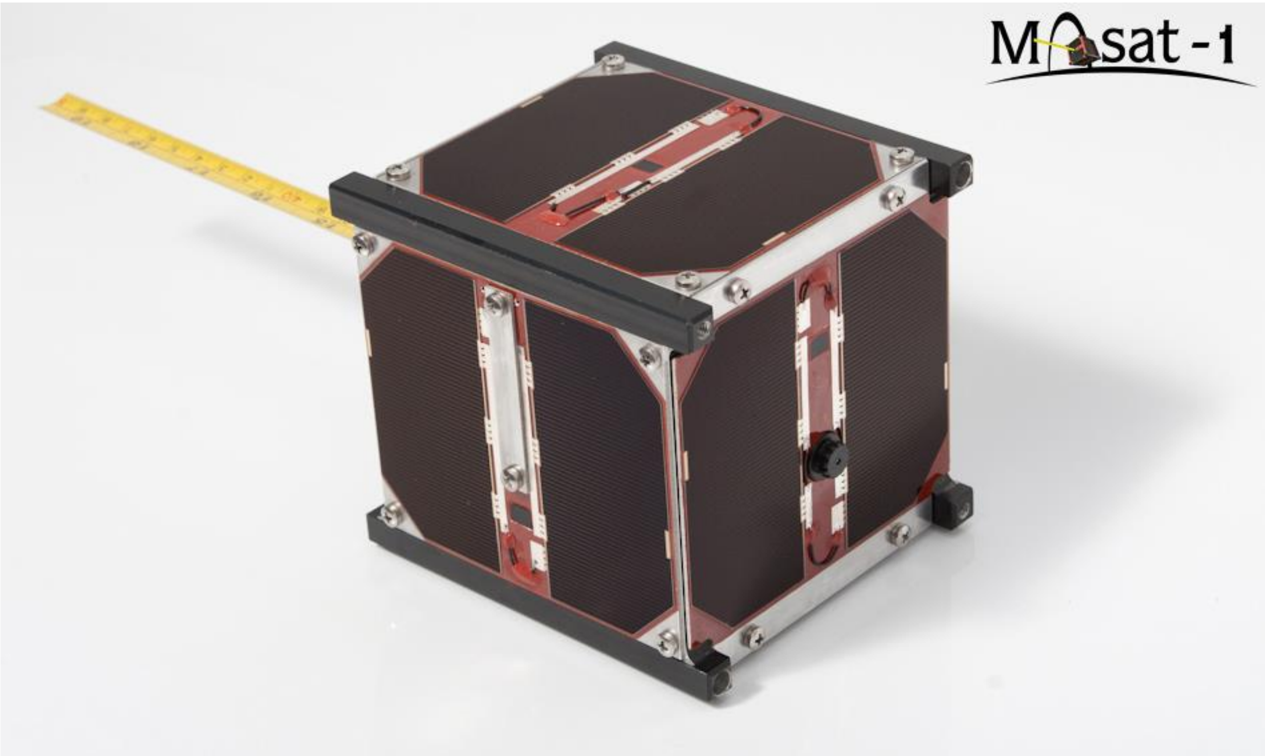
e) The total luminosity of the Coma cluster (in solar luminosity, $L_\odot$) is $L \approx 5 \times 10^{12} \mathrm{L}_\odot$. Calculate the mass-luminosity ratio of the cluster in solar mass per solar luminosity units. (2 p)

f) Which of the following statements is true (more than one answer is possible)? **Mark the letter of your answer with × on the answer sheet.** (4 p)

(A) The mass-luminosity ratio of the Coma cluster is much higher than that of a typical spiral galaxy, like the Milky Way.
(B) The mass-luminosity ratio of the Coma cluster is similar to that of a typical spiral galaxy.
(C) The mass-luminosity ratio of the Coma cluster is much less than that of a typical spiral galaxy.
(D) The Coma cluster contains much more dark matter than a typical spiral galaxy.
(E) The Coma cluster contains much less dark matter than a typical spiral galaxy.

14. Photographing a nanosatellite **(60 p)**

The very first, entirely Hungarian-made, satellite was "MASAT–1", a nanosatellite "cubesat". It was made mostly of aluminium with total mass 1 kg, sides length l = 10 cm and a longer communication aerial. It was designed and prepared at the Technical University of Budapest (BME) in 2009 by students. The launch occurred on February 13, 2012, using a Vega rocket from the Kourou launch site – together with several other cubesats from other countries. It operated successfully right up to the last minutes of its lifetime (the transmitted last data packages were captured by radio receivers on the evening of January 9, 2015, a few hours later the nanosatellite re-entered the atmosphere and disintegrated).



The orbital altitude of MASAT–1 changed between $h_{\min} = 350$ km and $h_{\max} = 1450$ km (due to its highly eccentric orbit), but in this question assume it to be in a circular orbit 900 km above the sea level.

The MASAT team wished to photograph their nanosatellite from the ground. Thus, they called the staff of Baja Astronomical Observatory (South Hungary, $\lambda_B = 19.010843°$, $\varphi_B = 46.180329°$, $h_B = 100$ m) to photograph the orbiting MASAT–1 with their telescope.

The observatory has a Ritchey-Chrétien type reflecting telescope with a diameter of 50 cm, and focal ratio of $f/8.4$. The CCD camera installed on the telescope had a 4096×4096 chip with 9 μm sized square pixels. The quantum efficiency of the CCD was about 70 %. The practical detection limit was about 19.5^m visual band applying a $\tau_{\exp} = 2$ min exposure time. Disregard any effects of seeing. The orbital inclination of MASAT–1 was about i=70° and the direction of the orbital motion was the same as the Earth rotation. In addition, let us assume that the reflection area always is 100 cm^2. Throughout this event, the telescope was pointed to the local zenith, and its RA motor was tracking the stars. Consider only light from the Sun, you can ignore light reflected from Earth and Moon. Ignore the effects of atmospheric extinction.

a) Calculate the apparent visual magnitude of this cubesat under ideal observational circumstances, i.e. when it was at the local zenith of the observing site (Baja, Hungary) at midnight. Omit all atmospheric effects, and consider the Earth to be a sphere. The albedo of a specular aluminium plate is $a \approx 0.70$. (10 p)

   Hint: Use a comparison of MASAT–1 with the Full Moon.

b) What was the Observatory's answer to the MASAT team; would it ever be possible to photograph their nanosatellite with the existing equipment at the observatory? **Mark "YES" or "NO" on the answer sheet.** Support your answer with detailed calculations. (42 p)

c) What would have been the answer if they had taken into account the blurring effect of the atmosphere, the so-called seeing? In Hungary the typical FWHM (Full Width at Half Maximum) value of the blurred stellar images – which can be approximated by a symmetric 2D Gaussian profile close to the zenith – is about 3.5″. **Mark "YES" or "NO" or "Close to the limit" on the answer sheet.** Support your answer with a short calculation. (8 p)

   Hint: Although the illumination of the seeing spot in the focal plane can be approximated by a symmetric 2D Gaussian profile, you can take it to be homogeneous in your calculation.

1. Photometry and spectroscopy of Nova Del 2013 **(60 p)**

Classical nova V339 Del (Nova Delphini 2013) was discovered by Koichi Itagaki at 6.8 magnitude on 14 August 2013 at 14:01 UT (MJD = 56518.584). Both professional and amateur astronomers analysed the photometry and spectroscopy of the nova. Less than 10 hours after the alert, when the night falls at the Piszkéstető Mountain Station of the Konkoly Observatory of the Hungarian Academy of Sciences, Hungarian astronomers took the first spectrum data of the nova using the eShel echelle spectrograph in the Gothard Astrophysical Observatory of Loránd Eötvös University attached to the 1 meter telescope of the Konkoly Observatory.

Refer to Fig 1.1 and Fig 1.2 to complete the questions. The larger versions of Fig 1.1 and Fig 1.2 are found on separate A3 papers.

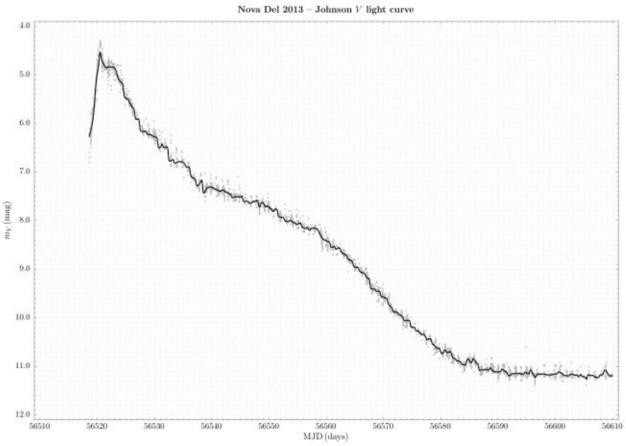


Fig. 1.1: Nova Del 2013 – Johnson V light curve

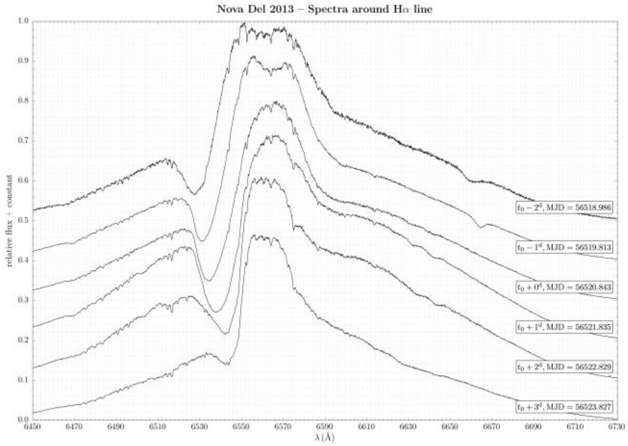


Fig. 1.2: Nova Del 2013 – Spectra around Hα line

Fig. 1.1 shows the visual light curve of the nova based on the data downloaded from the website of AAVSO (American Association of Variable Star Observers). On the horizontal and vertical axes, the Modified Julian Date (MJD = JD−2 400 000.5) of the observations and the Johnson V magnitudes are plotted, respectively. The grey circles (about 38000 data points) represent the measured values, while the continuous black line is the result of smoothing the data with a Gaussian filter (Full Width at Half Maximum = 0.5 day) to define an "average" light curve from the data points.

The rate of decline can be characterized by the values t_2 and t_3, which show the time interval in days in which a nova fades from its maximum brightness by 2 and 3 magnitudes.

A few empirical formulae between the peak of the absolute magnitude in the V band (M_0) and t_2, t_3 values can be found in the following literature:

(a) $M_0 = -7.92 - 0.81 \arctan \frac{1.32 - \log t_2}{0.23}$ (Della Valle, M. & Livio, M.: 1995, *ApJ* **452**, 704)

(b) $M_0 = -11.32 + 2.55 \log t_2$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

(c) $M_0 = -11.99 + 2.54 \log t_3$ (Downes, R.A. & Durbeck, H.W.: 2000, *AJ* **120**, 2007)

The $E(B-V)$ color excess of Nova Del 2013 (Chochol, D. et al.: 2014, *Contrib. Astron. Obs. Skalnaté Pleso* **43**, 330) is:

$$E(B-V) = 0.184 \pm 0.035$$

Fig 1.2 shows the nova spectra taken in the wavelength region around the Hα line on six consecutive nights before and after the time of the maximum brightness (t_0). The individual spectra have been shifted vertically for clarity. The Modified Julian Dates (MJD) of the observations are listed on the right hand side of each spectrum slice.

The Hα line shows the so-called P Cygni profile with very broad wings, which is typical not of novae only, but is present in almost all spectral types and is a reliable sign of a massive radial motion of matter ejected from the star. The P Cygni profile is composed of a strong, broad emission peak which is considered to be centered at the rest wavelength in air λ_0 of the line – for Hα $\lambda_0 = 6562.82$ Å – and a usually weaker, blueshifted absorption component. The expansion (radial) velocity of the shell can be approximated from the measured wavelength λ of the absorption peak using the well known Doppler formula connecting the displacement $\Delta\lambda = \lambda - \lambda_0$, the radial velocity v_r, and c, the speed of light.

Assume that the Hα line showing P Cygni profile is excited in the outermost part of the spherically expanding shell, and its extent at the moment of taking the first spectrum was still negligible.

a) From Fig. 1.1, estimate the Modified Julian Date of the peak magnitude (MJD_0) and the value of the peak magnitude itself. Consider the error of this brightness value to be 0.05^m. (3 p)

b) Estimate the Modified Julian Dates based on the time interval (days) in which the nova has faded by 2 and 3 magnitudes, then calculate t_2 and t_3 values. (6 p)

c) With reference to t_2 and t_3 from (b), determine the peak absolute magnitude of the nova using all three empirical formulae listed earlier, calculate their mean (M_0) and their standard deviation, and consider this latter as the uncertainty of M_0. (5 p)

The formula for standard deviation:

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \bar{x})^2}{n-1}}$$

d) Using the value of the color excess $E(B-V)$, determine the interstellar extinction A_V and its uncertainty in the direction of the nova. Use $R_V = 3.1$, without error. (4 p)

e) Estimate the distance to the nova and its uncertainty. Give the result in kpc. (11 p)

f) Measure the central wavelengths of the P Cygni absorption features plotted in Fig. 1.2 (refer to magnified version), and calculate the corresponding radial velocities. No error estimation is needed. (14 p)

g) Plot these radial velocities against the Modified Julian Dates of the observations. (6 p)

h) From the graph in (g), estimate the physical radius of the envelope at the end of the time interval. Give the answer in astronomical units (au). (7 p)

i) Knowing the distance to the nova and the physical radius of the spherical envelope 5 days after the discovery, estimate the apparent angular diameter of the envelope then. (4 p)

2. Triply eclipsing hierarchical triple stellar system (90 p)

HD 181068 was one of the brightest targets which was continuously observed during the almost 4-year-long primary mission of NASA's exoplanet-hunter *Kepler* space telescope. The spacecraft observed $\approx 3-4\times 10^{-3}$ magnitude dimmings every 0.453 days. (Note: The even dimmings were slightly smaller amplitude than the odd ones.) Furthermore, additional 0.007 magnitude, 2.3-day-long dimmings were detected every 22.7 days.

The correct explanation of this very unusual photometric behaviour was given by Hungarian astronomers. They found that HD 181068 is a compact hierarchical triple stellar system seen almost edge-on.

Hierarchical triple star systems consist of three stars; A, B, and C. Two of these stars (B and C) form an inner or close stellar binary system, whilst the outer component (star A) orbits at a distance from the inner system significantly larger (usually orders of magnitude) than the semi-major axis of the inner system. The schematic view of an example of a triple star system is illustrated in Fig. 2.1.

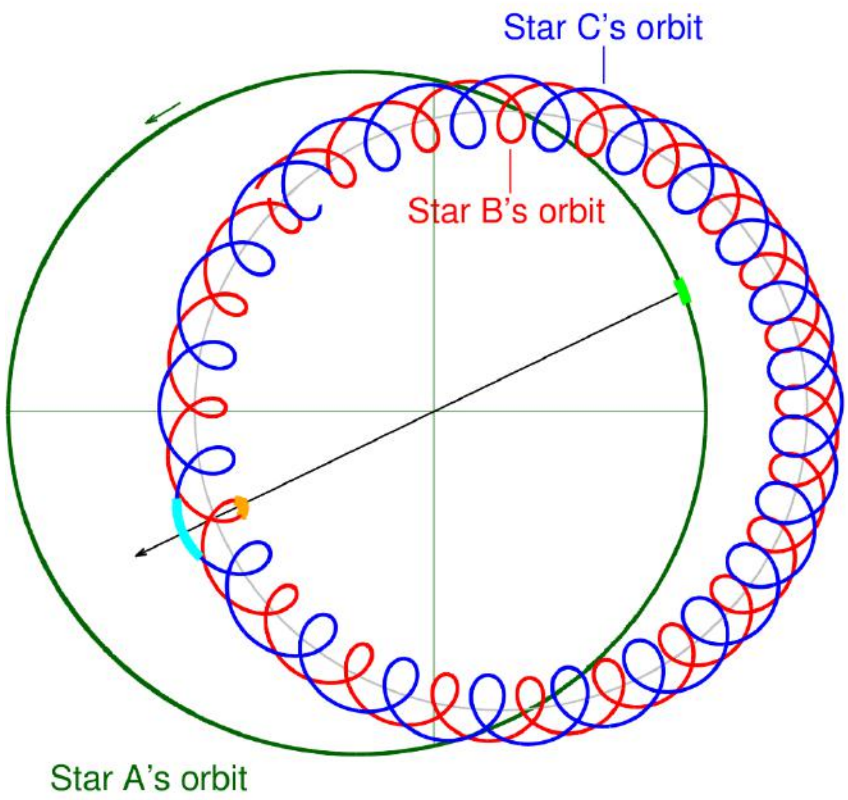


Fig. 2.1: The schematic pole-on view of a hypothetical , hierarchical triple stellar system. The black arrow is directed towards the Earth. The thick segments of the three orbits represent the stars' orbital arcs during an outer eclipse.

Mathematically, the motion of a hierarchical triple system can be well approximated with two unperturbed Keplerian two-body motions; (1) Keplerian motion of the inner binary. (2) The centre of mass of this close binary and the third star revolves on a second Keplerian orbit, "outer binary".

In this problem, stars B and C form a P_1 = 0.9056768-day-period eclipsing binary, while the centre of mass of these stars with star A forms the P_2 = 45.4711-day-period outer binary. As the orbital plane of this outer orbit is seen almost edge-on from the Kepler spacecraft (and from the Earth), during their revolution on the outer orbit, stars B and C eclipse not only each other, but also star A or, a half outer revolution later are eclipsed by it, causing the extra dimmings.

i. *Determination of the physical stellar sizes (and other quantities) from the geometry of the eclipses*

These assumptions are used throughout this section: (1) both the inner and outer orbits are exactly circular, (2) the orbital planes of the inner and outer orbits are identical, and (3) this plane is seen exactly edge-on (i.e. $i_1 = i_2 = 90°$ and $i_{\rm rel} = 0°$). Let's consider the extra dimmings, which are central eclipses (i.e. either occultations or transits – annular eclipses), therefore, these events have four contacts. In the case of an ordinary eclipsing binary (or of a transiting exoplanet) at the times of the outer contacts the sky-projected disks of the two objects connect with each other at one point from outside, while at the inner contacts they approach each other from inside. While this last statement is also valid for outer eclipses, the situation becomes more complex, because instead of two, three stars are involved into the eclipses. However, despite this fact, we can certainly define the times of each contact from the light curve, and furthermore, we can also decide explicitly which member of the inner binary is involved in a given contact. (The other member, of course, is always star A.)

In the table below, the accurate times of some contacts of different eclipses observed by the Kepler spacecraft, the contact types and the stars are documented. Time is expressed in barycentric Julian Days (BJD).

event no.	contact	stars	BJD	φ_1	φ_2
1	I	A, B	2455476.1096		
	II	A, C	2455476.4245		
	III	A, B	2455477.9677		
	IV	A, B	2455478.4722		
2	I	A, B	2455521.5217		
3	III	A, C	2455568.9434		
4	I	A, C	2455612.4733		
	III	A, C	2455614.3571		
5	III	A, B	2455659.9241		
	IV	A, C	2455660.2422		

a) Given that $T_{01} = 2455051.2361$ and $T_{02} = 2455522.7318$ denote the time of an inferior conjunction of the inner and outer binaries respectively (i.e. that time, when, from the perspective of the observer, star C eclipses star B, and when star A eclipses the centre of mass of stars B and C.)

Define

$\varphi_1(t) = \{(t - T_{01})/P_1\}$, and $\varphi_2(t) = \{(t - T_{02})/P_2\}$

as the photometric phases of the inner and outer binaries respectively. $\{x\}$ denotes the decimal part of the real number x. If $\{x\} < 0$, use $\{x\} + 1$ instead. Calculate the phases for the times of the tabulated contact events and **write the answers in the appropriate columns of the table on the answer sheet**. Round your answers to four decimal places. (10 p)

b) Determine, whether star A, or the close binary (i.e. stars B and C) were closer to the observer during each eclipsing event. **Write your answer in the table on the answer sheet.** (5 p)

c) Using the table above, calculate (1) the dimensionless radius of each star relative to the semi-major axis of the outer orbit ($R_{\rm A,B,C}/a_2$), (2) the ratio of the semi-major axes of the two orbits (a_1/a_2) and (3) the mass ratio of stars B and C ($q_1 = m_C/m_B$). *Hint:* Use at least four decimal place accuracy in your calculations. Be cautious, it may not be possible to use all theoretically possible contact combinations with a given limited accuracy of time data. (30 p)

d) Based on the results obtained above, calculate the outer mass ratio ($q_2 = m_{BC}/m_A$). (8 p)

ii. *Dynamical determination of the stellar masses using radial velocity (RV) and eclipse timing variations (ETV) measurements*

To obtain RV data, ground-based spectroscopic follow up observations were carried out with four different instruments. Only the lines of star A were detectable in all spectra. Plotting all the measurements against time, the RV curve was nicely fitted in the following form:

$$V_{\mathrm{rad,A}} = V_\gamma + K_A \sin\phi_{\mathrm{RV}},$$

where V_γ is the systemic velocity and K_A is the velocity amplitude:

$$V_\gamma = 6.993 \pm 0.011\ \mathrm{km\ s^{-1}},\ \ K_A = 37.195 \pm 0.053\ \mathrm{km\ s^{-1}},$$

$$P_2 = 45.4711 \pm 0.0002\ \mathrm{d},\ \ \phi_{\mathrm{RV}} = \frac{2\pi}{P_2}[t - (2455522.7318 \pm 0.0095)].$$

Furthermore, the researchers determined the mid-times of the regular eclipses of the close binary (formed by stars B and C), and found that the occurrence of for example the eclipsing minima belonging to the N^{th} orbital revolution can be described by the simple expression:

$$T_N = T_0 + P_1 N + A_{\mathrm{ETV}} \sin\left(\frac{2\pi}{P_2} P_1 N + \phi_0\right),$$

where

$$T_0 = \mathrm{BJD}\ 2455051.23607 \pm 5 \times 10^{-5},\ \ P_1 = 0.9056768 \pm 3 \times 10^{-7}\ \mathrm{d},$$

$$A_{\mathrm{ETV}} = 0.001446 \pm 0.000110\ \mathrm{d},\ \ \phi_0 = -0.76779 \pm 0.01937\ \mathrm{rad}.$$

In this expression A_{ETV} is the amplitude of the eclipse timing variation, T_0 denotes the mid-eclipse time of the reference (zeroth) primary eclipse, and N is the cycle number, which is an integer for primary eclipses (i.e. when the slightly fainter star C eclipses star B), and half-integer for secondary ones (i.e. when star B eclipses star C).

Determine (1) again the mass ratio ($q_2 = m_{BC}/m_A$) of the centre of mass of the inner binary and star A using only the results obtained in point ii., (2) the mass of component A (m_A) and (3) the total mass of the inner, close binary (m_{BC}). Calculate the errors for (1), (2), and (3) in masses. *Hint:* You can save much time by expressing the masses in solar mass and the orbital separations either in solar radius or au. (22 p)

iii. Using results obtained in questions 1 and 2, determine the masses of stars B and C respectively and calculate the physical dimensions of all three stars (i.e. stellar radii in physical units). (15 p)

OBSERVATIONAL ROUND – QUESTION SHEET

Telescope: 150/750 Newton
Eyepieces: 25 mm, 10 mm, Barlow lens: 2x

Note:

- Telescope is already polar aligned.
- In case of bad sky conditions at low altitudes, task 1 and 2 will be replaced by alternative tasks 1 and 2 (see page 3). In this situation, the telescope assistant will cross out Tasks 1 and 2.
- You have to use 25 mm eyepiece for tasks 1, 3 and 4.
- For these tasks, if you finish before the allotted time, you must keep tracking the object with the telescope till the end of allotted time. The telescope assistant will check the object only at the end of the allotted time.
- For task 2, we recommend using 10 mm eyepiece and Barlow 2x.
- For task 5, you are not allowed to use the telescope.

TASK 1: FINDERSCOPE ALIGNMENT

available time: 5 minutes 5 points

- The finderscope is NOT aligned at the beginning. **Point the telescope to Saturn and align the finderscope parallel to the main tube.**

If the alignment of Saturn is not within the crosshair of the finderscope, the telescope assistant will correct it – and you receive only partial or no points.

TASK 2: OBSERVATION OF SATURN

available time: 10 minutes 15 points

- In the upper box, the circle represents the disk of Saturn and the horizontal line is the E-W direction on the sky. Pay attention to direction of North (see top right corner).
  Mark position of Titan by a cross.

- The smaller box on the bottom right corner of first box is for drawing the rings of Saturn. Again the circle represents the disk of Saturn.
  Draw the rings of Saturn in this box with the correct size and orientation.
  Both the outer and inner edges of the ring are necessary, no faint ring details or gaps are needed.
  Keep orientation of the image the same as the orientation in the upper box.

- **Estimate the angular distance (in arcsec) and position angle (in degrees) of Titan relative to the center of Saturn.** You may do your calculations besides the answer.

Apparent major axis of the ring: 43"

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TASK 3: M57 – FIELD OF VIEW

available time: 10 minutes total: 10 points

- **Find the planetary nebula M57 (in constellation Lyra), and put it in the centre of the field of view in the main scope.**
- The star chart in the answer sheet shows a part of constellation Lyra. In this chart, **draw the FOV circle around M57 as accurately as possible.**

If you cannot find M57, the assistant will help you, but only after 5 minutes. In this case you will lose the marks for pointing to the object.

TASK 4: VARIABLE STAR – AF CYGNI

available time: 15 minutes total: 15 points

- **Use the given charts of the constellation Cygnus to find the variable star AF Cyg.**
  The large scale finder chart has normal orientation (N is up E is to the left)
  The smaller scale chart has 'telescope' orientation (S is up W is to the left)
  Brightness of reference stars are given without decimal points. e.g. '97' means 9.7 magnitude
  If you do not find AF Cyg, the telescope assistant cannot help you to point to it in this task.

- **Estimate the magnitude of AF Cyg by comparing it with the reference stars** and write it down, with decimal point, at one decimal accuracy (i.e. 9.7).
  Write the time of your observation in UTC. You may ask telescope assistant for the time in the local time zone (CEST).

TASK 5: NAKED EYE BRIGHTNESS ESTIMATION

available time: 5 minutes 5 points

- **Estimate the visual magnitude of the two naked-eye stars marked on the stellar chart of constellation Ursa Minor:**
  a) ζ UMi (zeta UMi = Alifa) – STAR 2
  b) γ UMi (gamma UMi = Pherkad) – STAR 1
  c) Write your estimate with one decimal accuracy (e.g. 8.6).
- **Estimate the angular distance between γ UMi (STAR 1) and Polaris in degrees.**

TASK 1/ ALTERNATIVE: **FINDERSCOPE ALIGNMENT**

available time: 5 minutes 5 points

- The finderscope is NOT aligned at the beginning. **Point the telescope to Altair (α Aql) and align the finderscope parallel to the main tube.**

If the alignment is not satisfactory, the telescope assistant will correct it – and you receive only partial or no points.

TASK 2 / ALTERNATIVE: **EPSILON LYRAE**

available time: 10 minutes 15 points

- **Find ε Lyr, and make a drawing of the field of view (with the object and other stars) with 10mm eyepiece.**
  Label the directions North and East by two arrows and mark them as 'N' and 'E'.
- **Estimate the angular distance between the wide pair (ε1-ε2), and estimate the position angle of the same pair.**
- **Increase the magnification with 2x Barlow lens to be able to resolve and separate the two close pairs. Estimate the angle** (in degrees to the nearest integer) **subtended by the two close pairs relative to each other.** (The enclosed angle of the two lines going through the two narrow pairs).
  Do not give any PA, only the relative angle of the two close pairs. No drawing is needed.

If you cannot find ε Lyr, the assistant can point to it for you, but only after 5 minutes. In this case you will lose the marks for pointing the telescope to the object.
The telescope assistant will check the object at the end of the 10 min limit. If you are ready sooner, keep the star in the FOV, and wait for the check.

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Student ID code: ☐☐☐–☐☐

PLANETARIUM ROUND – ANSWER SHEET

3 projected images with questions. Each part is 15 minutes long. Total time: 45 minutes.

PROBLEM 1

This is the sky above Keszthely at midnight. The projected sky does not show any Solar System objects.

QUESTIONS / TASKS:

1.1. There are 3 novae on the projected sky at 2nd magnitude. Mark their positions by circles on the star chart. (Please circle only 3 stars. If there are more than 3 circles, then each one in a wrong location will result in 1 point deduction.) [ ]

1.2. The Messier objects have been removed from the star chart given you. Mark all the Messier list globular clusters present in the projected sky on the star chart using crosses (X) and write the Messier number of each object near the cross marks. [ ]

1.3. The projected sky corresponds to the second half of which month (at midnight CEST) in Keszthely? Circle the correct month.

JAN / FEB / MAR / APR / MAY / JUN / JUL / AUG / SEP / OCT / NOV / DEC [ ]

1.4. What is the local sidereal time? (To an accuracy of 15 minutes.)

……………………………… [ ]

1.5. List six zodiacal constellations, which are partly or entirely visible. (Use the official IAU abbreviations or IAU designation. Every constellation named which is not visible in the projected sky will result in 1 point deduction.)

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[ ]

PROBLEM 2

We are standing somewhere on the Earth. The projected sky does not show any Solar System objects.

QUESTIONS / TASKS:

2.1. Determine the geographical latitude of this observing site: …………….° [ ]
In which hemisphere is the site situated? N / S (Circle the right one.) [ ]

2.2. Determine the azimuth of the 3 brightest stars on the projected sky. Azimuth is measured from North towards the East. Write the name of these stars in English or using their Bayer designation and their azimuths in the list below.

Bright star / name: ………………………………..…… Az: ………………° [ ]

Bright star / name: ………………………………..…… Az: ………………° [ ]

Bright star / name: ………………………………..…… Az: ………………° [ ]

2.3. Yellow × signs show the position of 3 comets. Which comet is closest to the ecliptic? (Circle the number below.)

1 / 2 / 3 [ ]

2.4. List nine constellations that contain circumpolar stars seen from the given observing site. (Use the official IAU abbreviations or IAU designation.)

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[ ]

2.5. Mintaka (δ Orionis) is setting at this moment. How many hours earlier did it rise? (To an accuracy of 15 minutes.)

……………… [ ]

PROBLEM 3

For this view, we are now standing on the Moon. At this instant viewed, the Earth is centrally eclipsing the Sun (see the red circle on the sky). Consequently the Moon is at one of its nodes now. Assume the longitudinal and latitudinal librations are exactly 0° at this moment.

QUESTIONS / TASKS:

3.1. At the time of this observation, which season is it in Hungary? (Circle the correct answer.)

Spring / Summer / Autumn / Winter [ ]

3.2. There is a yellow circle on the projected sky (next to the red circle), which denotes minor planet Juno, which is at a distance of exactly 3 au from the Sun at this moment. Estimate its distance to the Moon at this instant. (Rounded to the nearest integer in units of million km.) Assume all orbits to be circular.

………………………………million km [ ]

3.3. Approximately how much time (in Earth days) after the projected event will …

…the Sun set at your observing site? ……………………………… [ ]

…the Earth set at your observing site? ……………………………… [ ]

3.4. Determine the Lunar (Selenographic) coordinates of this observing site (as defined in the lunar map on the next page):

……………………………………. [ ]

What is the name of the large surface lunar area, where your observing site is situated? Do not use your national language, please use the official IAU nomenclature.
(See lunar map on next page.)

……………………………… [ ]

3.5. Estimate the distance from the observing site to the Apollo-11 landing site (0.6875 N, 23.4333 E):

……………………………… km [ ]