(D1) Calibrating distance ladder to the LMC [75 marks]

An accurate trigonometric parallax calibration for Galactic Cepheids has been long sought, but very difficult to achieve in practice. All known classical (Galactic) Cepheids are more than 250 pc away, therefore for direct distance estimates to have an uncertainty of up to 10%, parallax accuracies with uncertainties of up to ±0.2 milliarcsec are needed, requiring space observations. The Hipparcos satellite reported parallaxes for 200 of the nearest Cepheids, but even the best of these had high uncertainties. Recent progress has come with the use of the Fine Guidance Sensor on HST with parallaxes (in many cases) accurate to better than ±10% were obtained for 10 Cepheids, spanning the period range from 3.7 to 35.6 days. These nearby Cepheids span a range of distances from about 300 to 560 pc.

The measured periods, P, average magnitudes in V, K and I bands are given in **Table 1** as well as the A_V and A_K for extinction in V and K bands, respectively. The measured parallax with its uncertainty are also given in milliarcsec (mas). All measured apparent magnitude has negligibly small uncertainty.

Table 1: Period and average apparent magnitude of 5 Galactic Cepheids with accurate parallax measurements

	P (day)	**<V> (mag)**	**<K> (mag)**	**A_V (mag)**	**A_K (mag)**	**<I> (mag)**	**parallax (mas)**	**error (mas)**
RT Aur	3.728	5.464	3.925	0.20	0.02	4.778	2.40	0.19
FF Aql	4.471	5.372	3.465	0.64	0.08	4.510	2.81	0.18
X Sgr	7.013	4.556	2.557	0.58	0.07	3.661	3.00	0.18
ζ Gem	10.151	3.911	2.097	0.06	0.01	3.085	2.78	0.18
l Car	35.551	3.732	1.071	0.52	0.06	2.557	2.01	0.20

(D1.1) The observed correlation between period of a Cepheid and its brightness is usually described by the so-called "Period-Luminosity (PL) relation" where $L \propto P^{\beta}$. In fact, such relation is normally expressed in terms of period with the absolute magnitude instead of luminosity. Hereafter, we shall refer to Period-Absolute magnitude as the conventionally named "PL relation".

Use the data given in Table 1 to plot a suitable linear graph in order to derive the Cepheid PL relation for V- and K-band. You should separately plot each graph on different piece of graph paper. Determine the slope of the linear line that best describe the linear relation of the data. (You may find the relation $\Delta(\log_{10} x) \approx \dfrac{\Delta x}{x \log_e 10}$ useful)

[39.5 Marks]

Solution:

For Cepheid variable the PL relation is

$\log L = \beta \log P + C$ [1 Mark]

and recall that $F = \frac{L}{4\pi d^2}$

so $\log L$ can be written in term of absolute magnitude, M_x

$\log L = \log F + 2\log d + \log 4\pi$

and

$-\frac{m}{2.5} = \log F - \log F_0$

subtract the 2 equations, we get

$2.5\log L = -m + 5\log d + C*$

And from definition of absolute magnitude,

$M = m + 5 - 5\log_{10} d_{pc}$

We therefore get the relation between L vs. M,

$2.5\log L = -M + C''$

Substitute in the PL relation, we get

$M = \beta' \log P + C'$ or via realising that ($\log \mathrm{L} \propto M$) [1 Mark]

where

$\beta' = -2.5\beta$

-If student plot log L vs P, he/she should get slope which is (standard answer slope)/(-2.5). We already tell the student about using Absolute magnitude instead of Luminosity therefore half of the total marks should be deducted beyond this point if the student performs all corresponding calculations correctly.

- If the student plot absolute magnitude in reverse order and get positive slope but correct value for the slope, full mark should be awarded for the related parts

Calculate Absolute magnitude (M_x) and uncertainties from parallax and data in Table 1 for V & K

Calculate distances and uncertainties from

$d_{pc}(\text{parsec}) \approx \frac{1AU}{\theta_{\text{parallax}}(\text{arcsec})}$, $\Delta d_{pc} = \frac{\Delta\theta}{\theta} \times d_{pc}$

[2 mark]

And then calculate absolute magnitude and its uncertainty using

$$M_x = m_x - A_x + 5 - 5\log_{10} d_{pc}$$

$$\Delta M_x = \frac{5}{d_{pc}\ln 10} \times \Delta d_{pc}$$

[2 mark]

- Calculate d_{pc}, Δd_{pc}, $\log P$ and M_x and its error (30 x 0.5mark=**15 Marks** total, **not including ΔM_K**) (if student does not explicitly write out values of d_{pc}, Δd_{pc} but **get the correct answer for M_x and its error**, he/she **should get full-mark for that parts**)

- Calculate ΔM_K or from realizing that $\Delta M_K = \Delta M_V$ **(0.5 Mark)**

	d_{pc}	Δd_{pc}	logP	M_V	ΔM_V	M_K	ΔM_K
RT Aur	416.6	32.	0.572	-2.83	0.17	-4.19	0.17
FF Aql	355.8	22.	0.650	-3.02	0.13	-4.37	0.13
X Sgr	333.3	20.	0.846	-3.63	0.13	-5.12	0.13
ζ Gem	359.7	23.	1.007	-3.92	0.14	-5.69	0.14
l Car	497.5	49.	1.551	-5.27	0.21	-7.47	0.21

Plot Graph $\log P$ vs absolute magnitudes

- 2 plots x (5 point + 5 error bars) x 0.5 mark = **10 marks**
- 2 plots x 2 axis (clearly labelled) x 0.5 mark= **2 marks**
- 2 plots x Draw a straight line through data points and error bars x 1 mark = **2 marks**

V-band,

Slope should be in the range

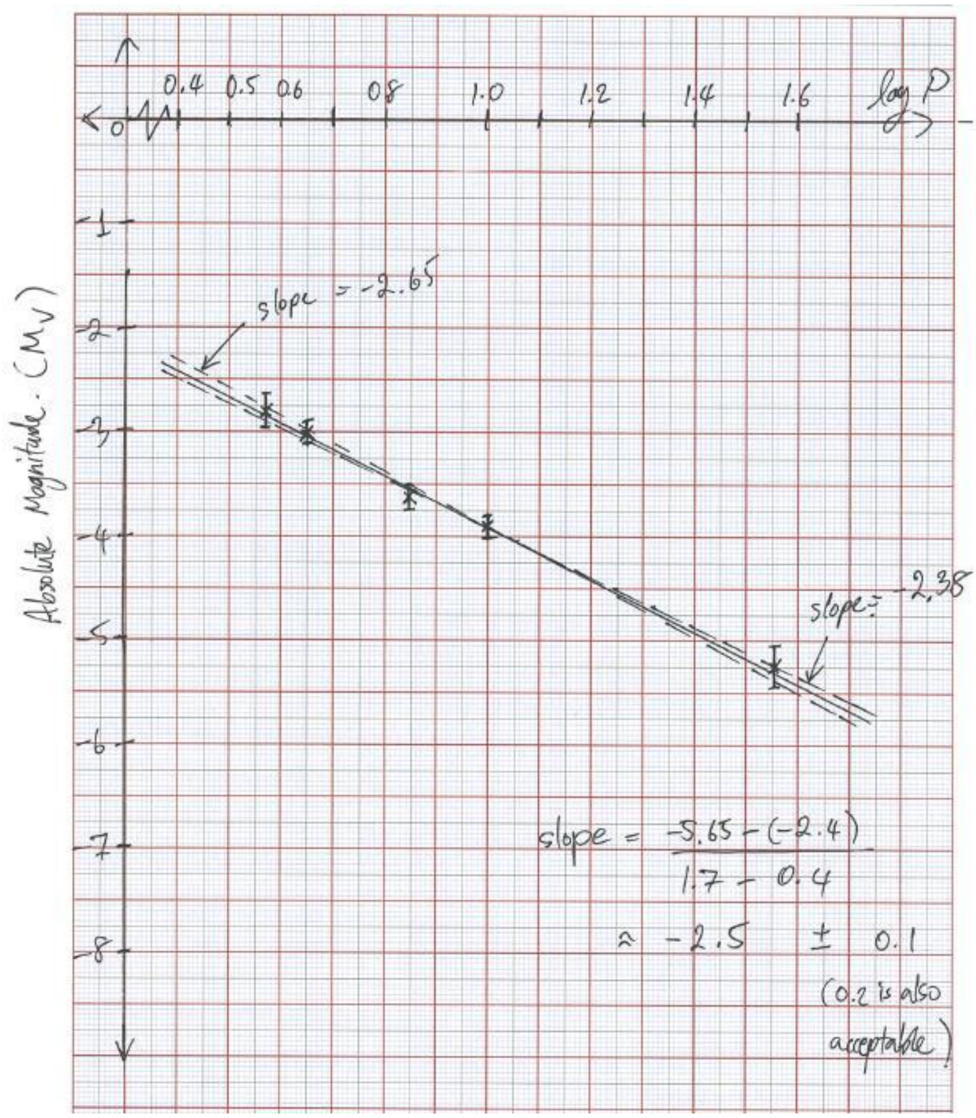
- **1 Mark** if answer within -2.5+/-0.2 (**half** if within -2.5+/-0.3)
- uncertainty should be between 0.1-0.3 (**1 mark**)

K-band,

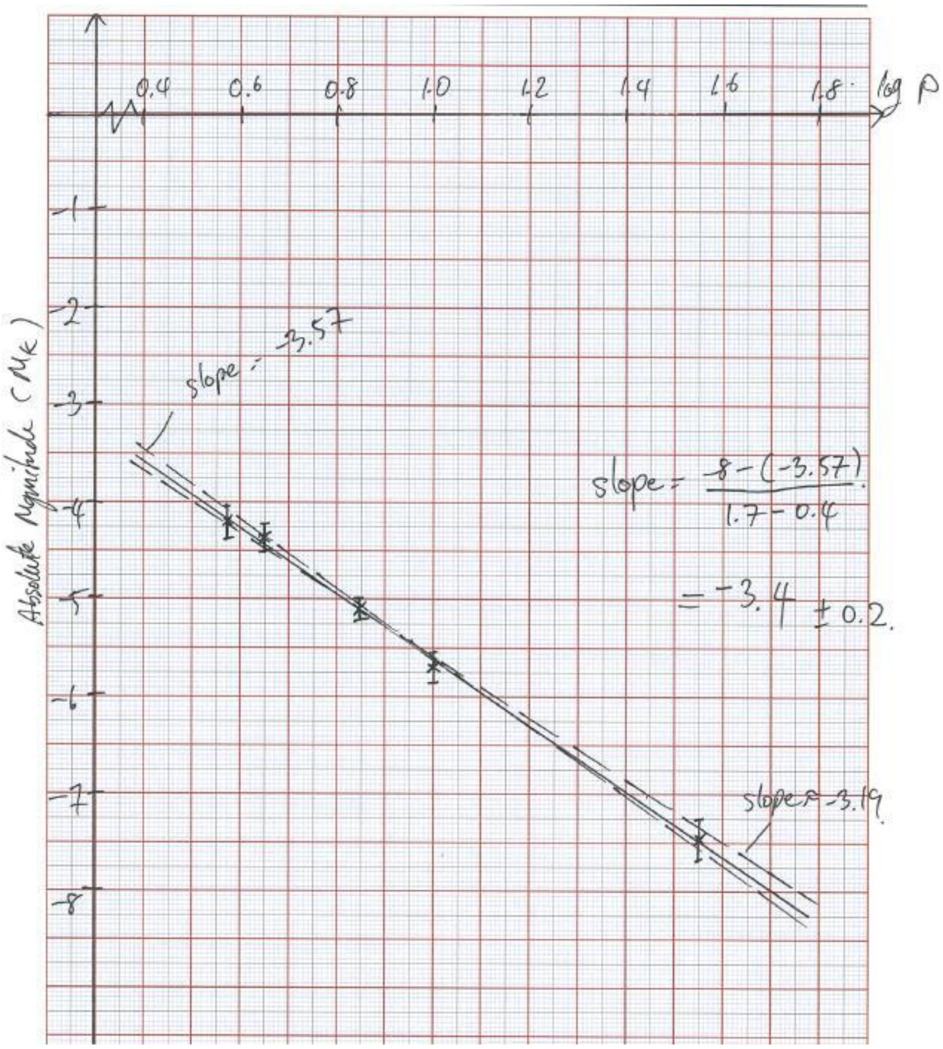
Slope should be in the range

- **1 Mark** if answer within -3.4+/-0.2 (**half** if within -3.4+/-0.3)
- estimated uncertainty should be between 0.1-0.3 (**1 mark**)

Plot for V-band data



Plot for K-band data



Any apparent differences in PL relations of stars in different bands can be explained if one also considers differences in colour. Therefore, the PL relation is in fact a PLC (Period-Luminosity-Colour) relation. This is due to reddening effect that causes extinction as a function of wavelength which can also vary among different Cepheids due to their different metallicities, foreground Inter Stellar Medium and dust.

A new reddening-free magnitude (or bandpass) called "Wesenheit" has been proposed which does not require the explicit information of the extinction of individual star but uses colour information of the star itself to get rid of the effect. For example, W_{VI} use V and I band photometry and is defined as

$$W_{VI} = V - \left[\frac{A_V}{E(V-I)}\right](V-I),$$

$$= V - R_v(V-I)$$

where R_v depends on the reddening law. In this case, we shall take the value of R_v to be 2.45.

(D1.2) From the data given in Table 1, plot and derive reddening-free PL relation using Wesenheit W_{VI} magnitude. Estimate the linear slope of the relation as well as its uncertainty. [14.5 Marks]

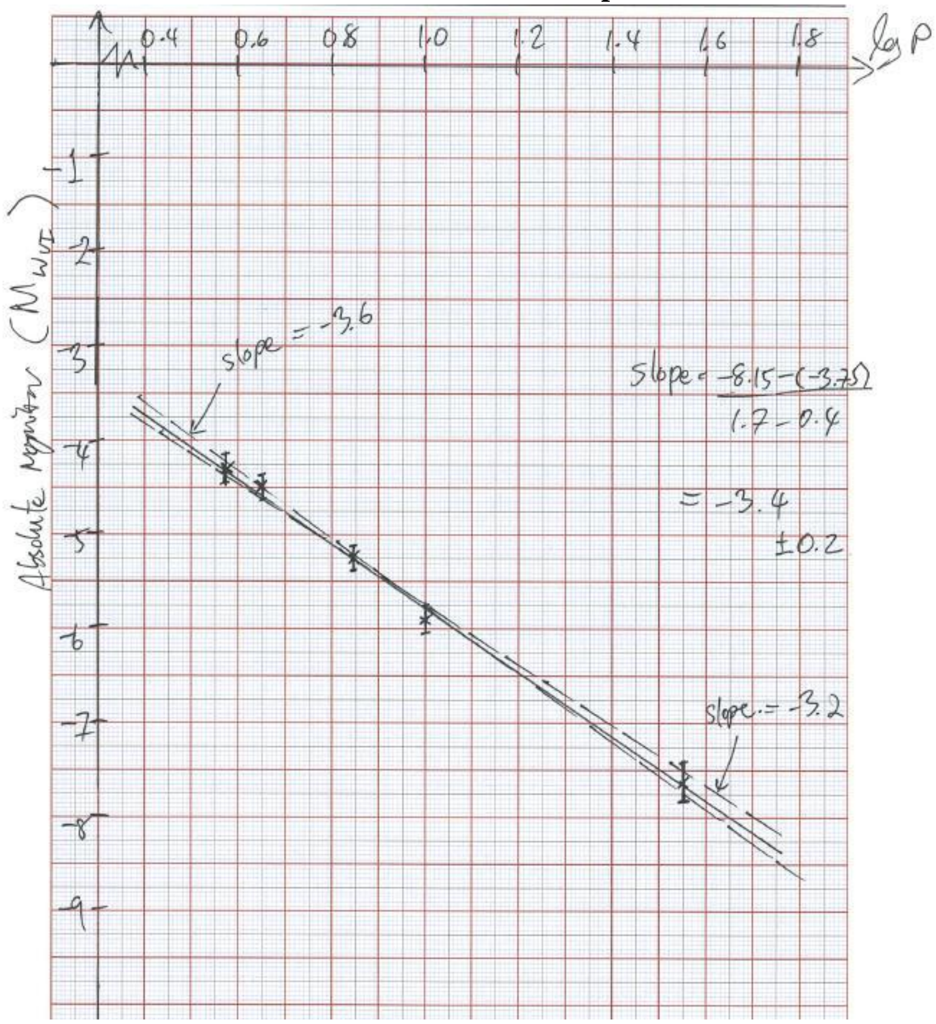
Solution:

- Calculate the Wesenheit magnitude from the given V, I and R_V values and then estimate the absolute magnitude for the Wesenheit system M_{WVI}

	W_{VI}	M_{WVI}
RT Aur	3.78	-4.31
FF Aql	3.26	-4.49
X Sgr	2.36	-5.25
ζ Gem	1.88	-5.89
l Car	0.85	-7.63

- calculate the values for W_{VI} and M_{WVI}, 10 values x 0.5 = **5 marks (not including** ΔM_{WVI})
- Realising that $\Delta M_{WVI} = \Delta M_V = \Delta M_K$ **or** by full calculation **0.5 mark**
- plot log *P* vs M_{WVI} x (5 point + 5 error bars))x 0.5 = **5 marks**)
- 2 axis (labels, units) x 0.5 = **1 mark**
- Draw appropriate straight line through the data or within error bar **1 mark**
- **1 Mark** for estimated slope within -3.4+/-0.2 (**half** if within -3.4+/-0.3)
- **1 Mark** for estimated uncertainty of slope between 0.1-0.3

Plot for W_{VI} bandpass



(D1.3) Next, we would like to use the newly derived PL relation from question (D1.1) & (D1.2) to estimate the distance to Large Magellenic Cloud (LMC) using periods and magnitudes of classical Cepheids in the LMC. In **table 2**, the periods, average extinction-corrected apparent magnitude**,** <V_{corr}>, and Wesenheit W_{VI} magnitudes are given.

Estimate distance modulus, μ, to each star and then use all the information to derive distance to LMC (in parsecs) and its standard deviation for each band. Compare if the derived distances are statistically different for the 2 bands (YES/NO). Are the standard deviations of the estimated distances for 2 bands different (YES/NO)? Based on this dataset, which band (V or Wesenheit) is more accurate in estimating the distance to LMC? [21 Marks]

Table 2: Period, average extinction-corrected apparent magnitude**,** <V_{corr}>, and average Wesenheit magnitude measurements of Cepheids in the LMC

	P (day)	<V_{corr}> mag	<W_{VI}> mag
HV12199	2.63	16.08	14.56
HV12203	2.95	15.93	14.40
HV12816	9.10	14.30	12.80
HV899	30.90	13.07	10.97
HV2257	39.36	12.86	10.54

Solution:

Distance modulus in each band is given by

$\mu_x = m_x - M_x$ [**0.5 Mark**]

And from previous section

$M_x = \beta'_x \log P + C'_x$,

where we have estimated slope β but not the intercept C'_x

Therefore, to get the distance in parsec we will need to evaluate

$\mu = m_x - (\beta_x \log P + C'_x) = 5\log_{10} d_{pc} - 5$ [**1 Mark**]

Using the best-estimate slope for each band and any point along the best-fit line (or intercept) student should be able to estimate C'_x

V-band: $C'_V = -1.4$ (**1 mark** if within +/-0.2, half if within +/-0.3)

W_{VI} band: $C'_{WVI} = -2.4$ (**1 mark** if within +/-0.2, half if within +/-0.3)

Star	**LogP**	μ_V	μ_{WVI}	**V distance (pc)**	**W_{VI} distance (pc)**
HV12199	0.42	18.53	18.39	50813	47596
HV12203	0.47	18.50	18.40	50223	47805
HV12816	0.96	18.10	18.46	41640	49221
HV899	1.49	18.19	18.43	43549	48660
HV2257	1.60	18.25	18.36	44620	47058
			Mean	46170	48068
			STD.	4116	865

- calculate the values ($\log P$, distance modulus, distance accurate to the nearest 100 pc)   25 x 0.5 mark=**12.5 marks**

- Calculate the mean (x 0.5 mark) and std. (x 0.5 mark) for each band (x2 bands) = **2 marks**

a) Answer **NO**, the distances estimated from V and Wesenheit are not statistically different **(1 mark)**

- full mark: In case of **"NO"** and distances estimated from V- and W_{VI}-band are within 1 standard deviation of one another

- full mark: In case of **"YES"** and distances estimated from V- and W_{VI}-band are not within 1 standard deviation of one another but due to wrong numerical calculation from previous part (already penalised)

- Zero mark: in all cases if the answer is not consistent with reasoning similar to above

b) Answer **YES**, standard deviation in the estimated distances from the 2 bands are significantly different **(1 mark)**

- full mark: if the answer is consistent with the calculated std. when the answer is either "YES" or "NO", "YES" if fractional uncertainty is similar and "NO" otherwise.

- zero mark: if the answer is not consistent with estimated uncertainties

c) Wessenheit magnitude is better at estimating the distance to LMC **(1 marks)**

- full mark: if answer is consistent with std.

- zero mark: otherwise

(D2) **The search for dark matter** **[75 marks]**

A low surface brightness galaxy (LSB) is a diffuse galaxy with a surface brightness that, when viewed from the Earth, is at least one magnitude lower than the ambient night sky.

Some of its matter is in the form of "baryonic" matter such as neutral hydrogen gas and stars. However, most of its matter is in the form of invisible mass which is so called "dark matter". In this question, we will investigate the mass of dark matter in a galaxy, the effect of dark matter on the rotation curves of the galaxy, and the distribution of dark matter in the galaxy.

The table below provides the data of a LSB galaxy named UGC4325. The galaxy is assumed to be edge-on. At every distance r from the centre of the galaxy, we measure

1. λ_{obs}, the observed wavelength of the Hα emission line. The Hubble expansion of the Universe has already been excluded from the data.

2. V_{gas}, the contribution of the gas component to the rotation due to M_{gas}, derived from HI surface densities.

3. V_*, the contribution of the stellar component to the rotation due to M_*, derived from R-band photometry.

The rotational velocities of the test particle due to the gas component, V_{gas}, and the star component, V_*, are defined as the velocities in the plane of the galaxy that would result from the corresponding components without any external influences. These velocities are calculated from the observed baryonic mass density distributions.

r (kpc)	λ_{obs} (nm)	V_{gas} (km/s)	V_* (km/s)
0.70	656.371	2.87	20.97
1.40	656.431	6.75	32.22
2.09	656.464	14.14	40.91
2.79	656.475	20.18	46.75
3.49	656.478	24.08	50.10
4.89	656.484	28.08	47.94
6.25	656.481	29.25	45.47
7.10	656.481	27.03	47.78
9.03	656.482	25.90	45.32
12.05	656.482	21.03	42.30

The mass of dark matter $M_{\text{DM}}(r)$ within a volume of radius r can be defined in terms of the rotational velocity due to dark matter V_{DM}, the radius r and gravitational constant G,

$$M_{\rm DM}(r) = \frac{rV_{\rm DM}^2}{G}. \qquad (1)$$

For the best estimate, the observed rotational velocity $V_{\rm obs}$ can be modeled as

$$V_{\rm obs}^2 = V_{\rm gas}^2 + V_*^2 + V_{\rm DM}^2. \qquad (2)$$

The observed rotational velocity $V_{\rm obs}$ depends on the mass of the galaxy $M(r)$ within a volume of radius r measured from the galaxy's centre.

The mass density $\rho_{\rm DM}(r)$ of dark matter within a volume of radius r can be modeled by a galaxy density profile,

$$\rho_{\rm DM}(r) = \frac{\rho_0}{1+\left(\frac{r}{r_{\rm C}}\right)^2} \qquad (3)$$

where ρ_0 and $r_{\rm C}$ are the central density and the core radius of the galaxy, respectively. According to the density profile, the mass of dark matter $M_{\rm DM}(r)$ within a volume of a radius r can be described by

$$M_{\rm DM}(r) = 4\pi\rho_0 r_{\rm C}^2\left[r - r_C \arctan(r/r_C)\right]. \qquad (4)$$

Part 1 The mass of dark matter and rotation curves of the galaxy

(D2.1) In laboratories on Earth, Hα has an emitted wavelength $\lambda_{\rm emit}$ of 656.281 nm. Compute the observed rotational velocities of the galaxy $V_{\rm obs}$ and the rotational velocities due to the dark matter $V_{\rm DM}$ at distance r in units of km/s.

For the different values of r given in the table, compute the dynamical mass $M(r)$ and the mass of dark matter $M_{\rm DM}(r)$ in solar masses. [21]

Solution:

Redshift: $z = \frac{\lambda_{\rm obs} - \lambda_{\rm emit}}{\lambda_{\rm emit}}$ [0.5]

Observed rotation velocity: $V_{\rm obs} = zc$ [0.5]

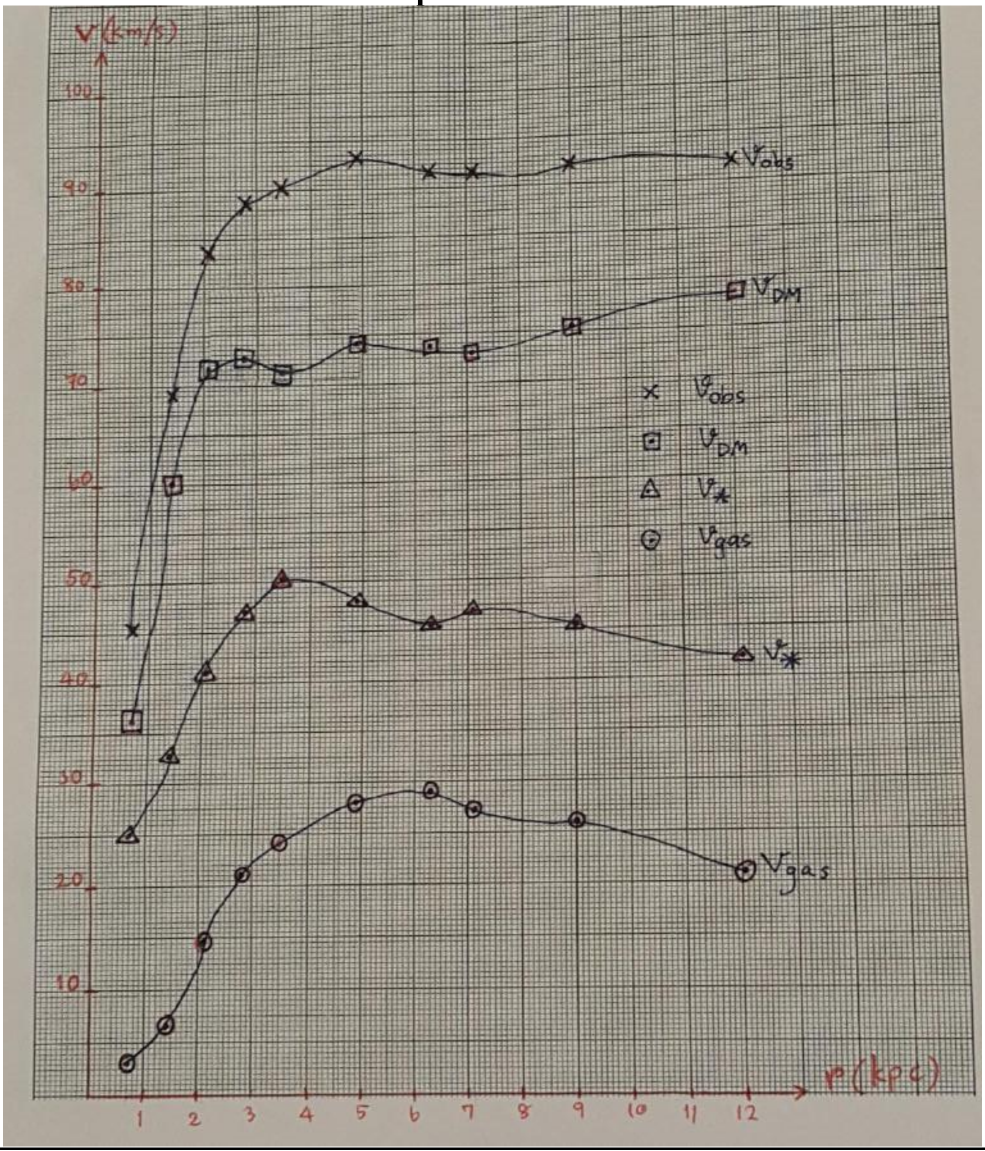
Values correctly computed: 20.0 (0.5 for each values)

r (kpc)	z	V_{obs} (km/s)	V_{DM} (km/s)	$M(r)$ $(M_\odot)$	$M_{DM}(r)$ $(M_\odot)$
0.70	$1.37\text{x}10^{-4}$	41.11	35.25	$2.75\text{x}10^{8}$	$2.02\text{x}10^{8}$
1.40	$2.29\text{ x}10^{-4}$	68.52	60.10	$1.53\text{x}10^{9}$	$1.18\text{x}10^{9}$
2.09	$2.79\text{ x}10^{-4}$	83.60	71.52	$3.40\text{x}10^{9}$	$2.49\text{x}10^{9}$
2.79	$2.96\text{ x}10^{-4}$	88.62	72.53	$5.09\text{x}10^{9}$	$3.41\text{x}10^{9}$
3.49	$3.00\text{ x}10^{-4}$	89.99	70.77	$6.57\text{x}10^{9}$	$4.06\text{x}10^{9}$
4.89	$3.09\text{ x}10^{-4}$	92.73	74.25	$9.78\text{x}10^{9}$	$6.27\text{x}10^{9}$
6.25	$3.05\text{ x}10^{-4}$	91.36	73.65	$1.21\text{x}10^{10}$	$7.88\text{x}10^{9}$
7.10	$3.05\text{ x}10^{-4}$	91.36	73.03	$1.38\text{ x}10^{10}$	$8.80\text{x}10^{9}$
9.03	$3.06\text{ x}10^{-4}$	91.82	75.54	$1.77\text{x}10^{10}$	$1.20\text{x}10^{10}$
12.05	$3.06\text{ x}10^{-4}$	91.82	78.74	$2.36\text{x}10^{10}$	$1.74\text{x}10^{10}$

(D2.2) Create rotation curves of the galaxy on graph paper by plotting the points of V_{obs}, V_{DM}, V_{gas}, V_* versus the radius r and draw smooth curves through the points (mark your graph as “D2.2”).

Order the contribution of the different components to the observed velocity in descending order. [16]

Graph Number D2.2



- Plot uses more than 50% of graph paper: 1.0
- Both axes labels (*V* and *r*) and their units presented: 2.0
- Ticks and values on axes (or scale written explicitly): 1.0
- All plots labels (V_{obs}, V_{DM}, V_{gas}, V_{*}) presented: 2.0, each label gets 0.5 mark
- Points correctly plotted: 4 (0.1 for each point)

  Values of V_{obs}, V_{DM} wrong due to wrong values of *z* gets a maximum of 4.0 marks
- Smooth curve through points: 4.0 (1.0 per curve)
- Order of contribution: 2.0, correct order only

(D2.3) Take a data points at small *r* and large *r* to estimate ρ_0 and r_C. Note that for large values of *x*, $\arctan(x) \approx \pi / 2$ and at small *x*, $\arctan(x) \approx x - x^3/3$. [7]

Solution

Evaluating for ρ_0:

$M_{DM}(r) = 4\pi\rho_0 r_C^2 [r - r_C \arctan(r/r_C)]$

$M_{DM}(r) = 4\pi\rho_0 r_C^3 [x - \arctan(x)]$, where $x = r/r_C$ [1.0]

$M_{DM}(r) \approx 4\pi\rho_0 r_C^3 \left[x - (x - \frac{x^3}{3})\right]$, for small x [1.0]

$$M_{DM}(r) \approx 4\pi\rho_0 r_C^3 \left(\frac{x^3}{3}\right) = \frac{4\pi\rho_0 r^3}{3}$$

$\rho_0 \approx \frac{2.02\times10^8 M_\odot \times 3}{4\pi(0.7kpc)^3} = 1.42\times10^8 M_\odot/kpc^3$ [1.5]

Selected correct data to put into the formula gets 0.5, correct answer gets 0.5 and correct unit gets 0.5.

$$M_{DM}(r) = 4\pi\rho_0 r_C^2 \left[r - r_C \arctan\left(\frac{r}{r_C}\right)\right]$$

Evaluating for r_C (Method 1):

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2 \left[r - r_C \frac{\pi}{2}\right]$, [1.0]

Take the last two data points at large r, then we get (for $r \gg r_C$

$\Delta M_{DM}(r) \approx 4\pi\rho_0 r_C^2 [\Delta r]$ [1.5]

$r_C = 1.01\ kpc$ [1.0]

Evaluating for r_C (Method 2):

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2 \left[r - r_C \frac{\pi}{2}\right]$, [1.0]

Take the last data point, we get a cubic equation which students can solve to give

$r_C = -0.855, 0.964, 7.56$ kpc [1.0]

Select the correct $r_C = 0.964$ for the final answer
because for the last data point $r \gg r_C$ [1.5]

Less accurate r_C :

Omit the last term,

$M_{DM}(r) \approx 4\pi\rho_0 r_C^2 r$ [1.0]

$r_C \approx \sqrt{\frac{1.74\times10^{10} M_\odot}{4\pi\times1.40\times\frac{10^8 M_\odot}{kpc^3}\times12.05kpc}} = 0.901\ kpc$ [1.0]

Part 2 Dark matter distribution

(D2.4) By comparing Equation (4) to a linear function, the central density ρ_0 could also be found by a linear fit. Plot an appropriate graph so that a linear fit can be used to find another value of ρ_0. Evaluate ρ_0 in units of $\frac{M_\odot}{kpc^3}$ (mark your graph as "D2.4"). If you cannot find the value of r_C from the previous part, use $r_C = 3.2\ \text{kpc}$ as an estimate for this part. [19]

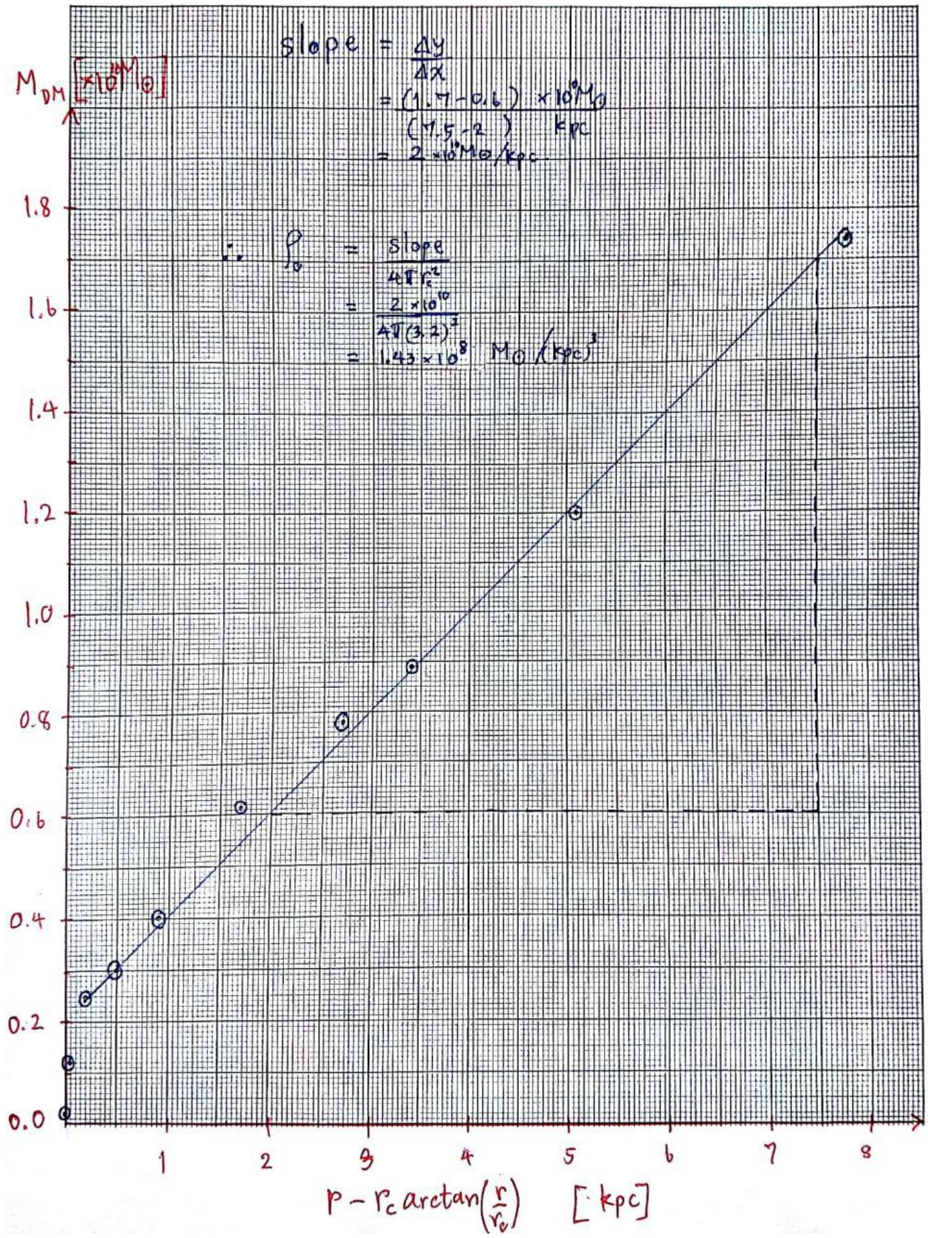
Solution 1:

$x = \left[r - r_c \arctan\left(\frac{r}{r_C}\right)\right]$

$y = M_{DM}(r)$

$x = \left[r - r_C \arctan(r / r_C)\right]$ (kpc)	$y = M_{DM}(r)$ (kg)
0.010855	2.02×10^{8}
0.0803	1.18×10^{9}
0.2386	2.49×10^{9}
0.495	3.41×10^{9}
0.838	4.06×10^{9}
1.718	6.27×10^{9}
2.738	7.88×10^{9}
3.428	8.80×10^{9}
5.093	1.20×10^{10}
7.854	1.74×10^{10}

Graph Number D2.4: Solution 1



- Choose correct axes: 1.0, each correct axis get 0.5 mark
- Values correctly computed: 8.0 (0.4 for each value, 20 data points)
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.2 for each point, 20 data points)
  Values of x axis wrong due to previous wrong values get a maximum of 2.0 marks
- Credit for good visual linear fit: 1.0
- Correct value of slope: 1.0
- Correct value of ρ_0 : 1.0

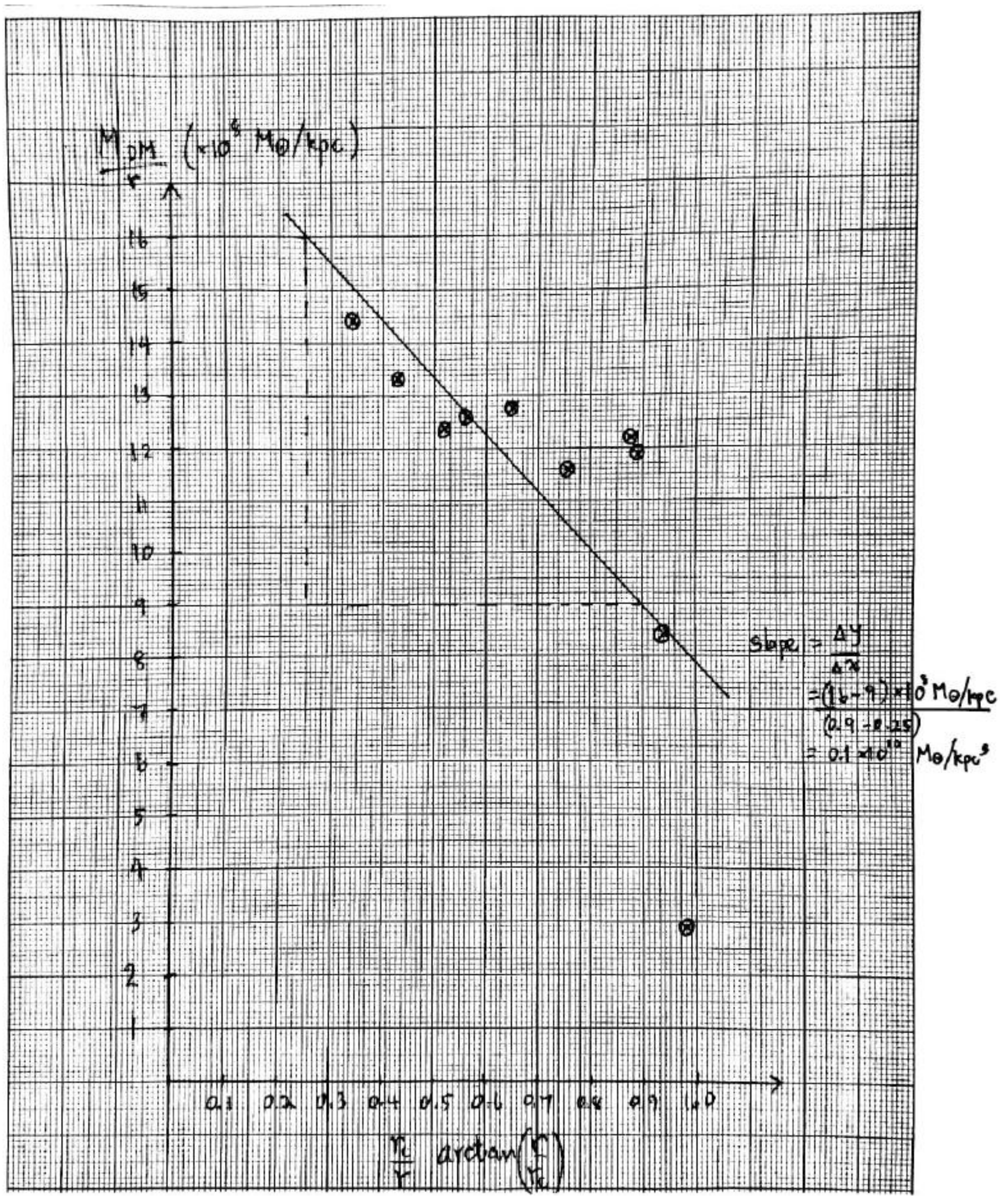
Solution 2:

$x = \frac{r_C}{r} \arctan\left(\frac{r}{r_C}\right)$

$y = M_{DM}(r)$

$x = \frac{r_C}{r} \arctan\left(\frac{r}{r_C}\right)$	$y = M_{\mathrm{DM}}(r)$ $(M_\odot)$
0.984	2.89×10^8
0.943	8.40×10^8
0.886	1.19×10^9
0.822	1.22×10^9
0.760	1.16×10^9
0.649	1.28×10^9
0.562	1.26×10^9
0.517	1.24×10^9
0.436	1.33×10^9
0.348	1.44×10^9

Graph Number D2.4: Solution 2



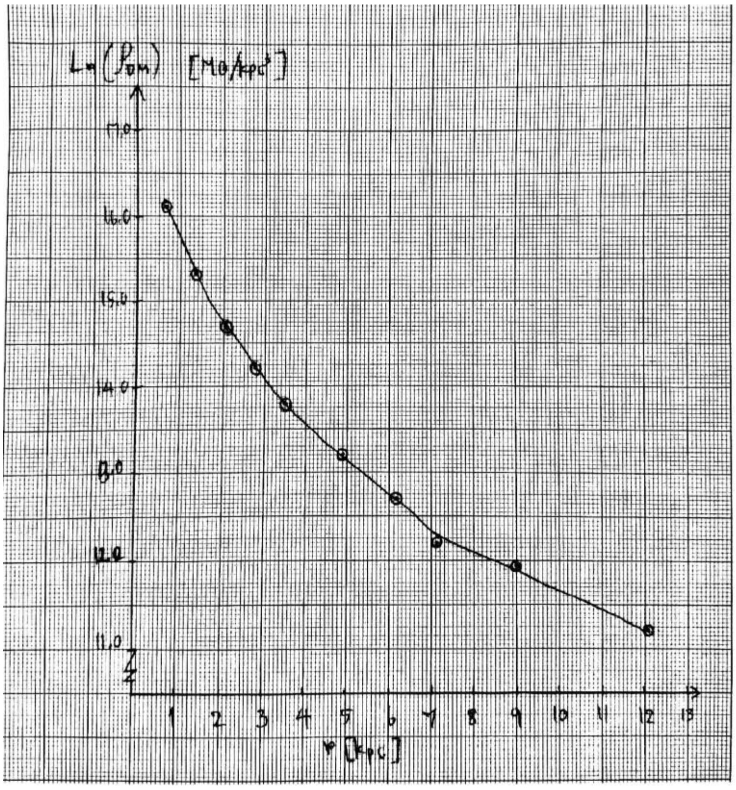
- Choose correct axes: 1.0, each correct axis get 0.5 mark
- Values correctly computed: 8.0 (0.4 for each value, 20 data points)
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.2 for each point, 20 data points)
  Values of x axis wrong due to previous wrong values gets a maximum of 2.0 marks
- Credit for good visual linear fit: 1.0
- Correct value of slope: 1.0
- Correct value of ρ_0 : 1.0

(D2.5) Compute logarithmic values of the dark matter density, ln [ρ_{DM}(r)], and plot the distribution of the dark matter in the galaxy as a function of radius r on graph paper (mark your graph as "D2.5"). [12]

Solution 1: $r_c = 0.906\ kpc$ from D2.3

r (kpc)	ln [ρ_{DM}(r)] ($\frac{M_\odot}{kpc^3}$)
0.70	16.11
1.40	15.36
2.09	14.73
2.79	14.23
3.49	13.81
4.89	13.17
6.25	12.69
7.10	12.44
9.03	11.97
12.05	11.40

Graph Number D2.5: Solution 1



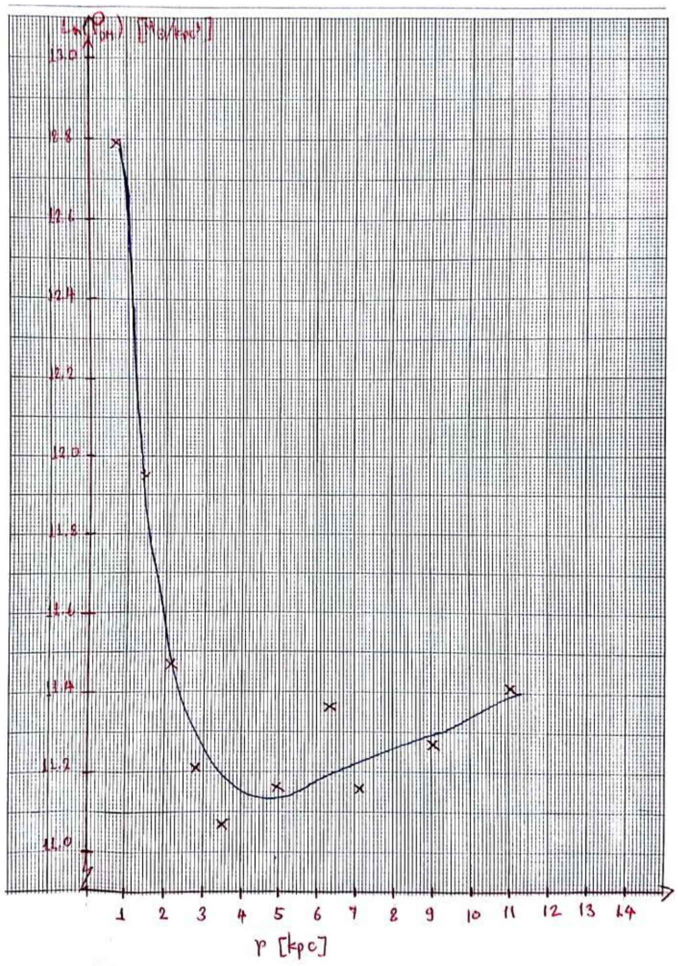
- Values correctly computed: 4.0 (0.4 for each values, 10 data points)

  Values of ln [ρ_{DM}(r)] wrong due to the wrong value of ρ_0 gets a maximum of 2.0 marks
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.4 for each point, 10 data points)
  Values of ln [ρ_{DM}(r)] wrong due to previous wrong values gets a maximum of 2.0 marks
- Smooth curve through points: 1.0

Solution 2: $r_c = 3.2\ kpc$

r (kpc)	ln [ρ_{DM}(r)] $(\frac{M_\odot}{kpc^3})$
0.70	12.79
1.40	11.95
2.09	11.47
2.79	11.21
3.49	11.07
4.89	11.16
6.25	11.26
7.10	11.16
9.03	11.27
12.05	11.41

Graph Number D2.5 Solution 2



- Values correctly computed: 4.0 (0.4 for each values, 10 data points)
  Values of ln [ρ_{DM}(r)] wrong due to the wrong value of ρ_0 gets a maximum of 2.0 marks
- Plot uses more than 50% of graph paper: 1.0
- Both axes labels presented: 1.0
- Ticks and values on axes (or scale written explicitly): 1.0
- Points correctly plotted: 4.0 (0.4 for each point, 10 data points)
  Values of ln [ρ_{DM}(r)] wrong due to previous wrong values gets a maximum of 2.0 marks
- Smooth curve through points: 1.0